**E7-1** $x_{\rm cm} = (7.36\times10^{22}{\rm kg})(3.82\times10^{8}{\rm m})/(7.36\times10^{22}{\rm kg} + 5.98\times10^{34}{\rm kg}) = 4.64\times10^{6}{\rm m}$. This is less than the radius of the Earth.

**E7-2** If the particles are $l$ apart then

$$x_1 = m_1 l(m_1 + m_2)$$

is the distance from particle 1 to the center of mass and

$$x_2 = m_2 l(m_1 + m_2)$$

is the distance from particle 2 to the center of mass. Divide the top equation by the bottom and

$$x_1/x_2 = m_1/m_2.$$

**E7-3** The center of mass velocity is given by Eq. 7-1,

$$\begin{aligned}\vec{\mathbf{v}}_{\rm cm} &= \frac{m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2}{m_1 + m_2},\\ &= \frac{(2210\ {\rm kg})(105\ {\rm km/h}) + (2080\ {\rm kg})(43.5\ {\rm km/h})}{(2210\ {\rm kg}) + (2080\ {\rm kg})} = 75.2\ {\rm km/h}.\end{aligned}$$

**E7-4** They will meet at the center of mass, so

$$x_{\rm cm} = (65\,{\rm kg})(9.7\,{\rm m})/(65\,{\rm kg} + 42\,{\rm kg}) = 5.9\,{\rm m}.$$

**E7-5** (a) No external forces, center of mass doesn't move.
(b) The collide at the center of mass,

$$x_{\rm cm} = (4.29\,{\rm kg})(1.64\,{\rm m})/(4.29\,{\rm kg} + 1.43\,{\rm kg}) = 1.23\,{\rm m}.$$

**E7-6** The range of the center of mass is

$$R = v_0^2 \sin 2\theta\,/g = (466\,{\rm m/s})^2 \sin(2\times 57.4^\circ)/(9.81\,{\rm m/s^2}) = 2.01\times10^4\,{\rm m}.$$

Half lands directly underneath the highest point, or $1.00\times10^4$ m. The other piece must land at $x$, such that

$$2.01\times10^4\,{\rm m} = (1.00\times10^4\,{\rm m} + x)/2;$$

then $x = 3.02\times10^4$ m.

**E7-7** The center of mass of the boat + dog doesn't move because there are no external forces on the system. Define the coordinate system so that distances are measured from the shore, so toward the shore is in the negative $x$ direction. The *change* in position of the center of mass is given by

$$\Delta x_{\rm cm} = \frac{m_d \Delta x_d + m_b \Delta x_b}{m_d + m_b} = 0,$$

Both $\Delta x_d$ and $\Delta x_b$ are measured with respect to the shore; we are given $\Delta x_{db} = -8.50$ ft, the displacement of the dog with respect to the boat. But

$$\Delta x_d = \Delta x_{db} + \Delta x_b.$$

Since we want to find out about the dog, we'll substitute for the boat's displacement,

$$0 = \frac{m_d \Delta x_d + m_b \left(\Delta x_d - \Delta x_{db}\right)}{m_d + m_b}.$$

Rearrange and solve for $\Delta x_d$. Use $W = mg$ and multiply the top and bottom of the expression by $g$. Then

$$\Delta x_d = \frac{m_b \Delta x_{db}}{m_d + m_b}\frac{g}{g} = \frac{(46.4 \text{ lb})(-8.50 \text{ ft})}{(10.8 \text{ lb}) + (46.4 \text{ lb})} = -6.90 \text{ ft}.$$

The dog is now $21.4 - 6.9 = 14.5$ feet from shore.

**E7-8** Richard has too much time on his hands.

The center of mass of the system is $x_{\text{cm}}$ away from the center of the boat. Switching seats is effectively the same thing as rotating the canoe through 180°, so the center of mass of the system has moved through a distance of $2x_{\text{cm}} = 0.412\,\text{m}$. Then $x_{\text{cm}} = 0.206\,\text{m}$. Then

$$x_{\text{cm}} = (Ml - ml)/(M + m + m_c) = 0.206\,\text{m},$$

where $l = 1.47\,\text{m}$, $M = 78.4\,\text{kg}$, $m_c = 31.6\,\text{kg}$, and $m$ is Judy's mass. Rearrange,

$$m = \frac{Ml - (M + m_c)x_{\text{cm}}}{l + x_{\text{cm}}} = \frac{(78.4\,\text{kg})(1.47\,\text{m}) - (78.4\,\text{kg} + 31.6\,\text{kg})(0.206\,\text{m})}{(1.47\,\text{m}) + (0.206\,\text{m})} = 55.2\,\text{kg}.$$

**E7-9** It takes the man $t = (18.2\,\text{m})/(2.08\,\text{m/s}) = 8.75\,\text{s}$ to walk to the front of the boat. During this time the center of mass of the system has moved forward $x = (4.16\,\text{m/s})(8.75\,\text{s}) = 36.4\,\text{m}$. But in walking forward to the front of the boat the man shifted the center of mass by a distance of $(84.4\,\text{kg})(18.2\,\text{m})/(84.4\,\text{kg} + 425\,\text{kg}) = 3.02\,\text{m}$, so the boat only traveled $36.4\,\text{m} - 3.02\,\text{m} = 33.4\,\text{m}$.

**E7-10** Do each coordinate separately.

$$x_{\text{cm}} = \frac{(3\,\text{kg})(0) + (8\,\text{kg})(1\,\text{m}) + (4\,\text{kg})(2\,\text{m})}{(3\,\text{kg}) + (8\,\text{kg}) + (4\,\text{kg})} = 1.07\,\text{m}$$

and

$$y_{\text{cm}} = \frac{(3\,\text{kg})(0) + (8\,\text{kg})(2\,\text{m}) + (4\,\text{kg})(1\,\text{m})}{(3\,\text{kg}) + (8\,\text{kg}) + (4\,\text{kg})} = 1.33\,\text{m}$$

$\boxed{\textbf{E7-11}}$ The center of mass of the three hydrogen atoms will be at the center of the pyramid base. The problem is then reduced to finding the center of mass of the nitrogen atom and the three hydrogen atom triangle. This molecular center of mass must lie on the dotted line in Fig. 7-27.

The location of the plane of the hydrogen atoms can be found from Pythagoras theorem

$$y_{\text{h}} = \sqrt{(10.14\times 10^{-11}\text{m})^2 - (9.40\times 10^{-11}\text{m})^2} = 3.8\times 10^{-11}\text{m}.$$

This distance can be used to find the center of mass of the molecule. From Eq. 7-2,

$$y_{\text{cm}} = \frac{m_{\text{n}} y_{\text{n}} + m_{\text{h}} y_{\text{h}}}{m_{\text{n}} + m_{\text{h}}} = \frac{(13.9 m_{\text{h}})(0) + (3 m_{\text{h}})(3.8\times 10^{-11}\text{m})}{(13.9 m_{\text{h}}) + (3 m_{\text{h}})} = 6.75\times 10^{-12}\text{m}.$$

**E7-12** The velocity components of the center of mass at $t = 1.42\,\text{s}$ are $v_{\text{cm},x} = 7.3\,\text{m/s}$ and $v_{\text{cm},y} = (10.0\,\text{m/s}) - (9.81\,\text{m/s})(1.42\,\text{s}) = -3.93\,\text{m/s}$. Then the velocity components of the "other" piece are

$$v_{2,x} = [(9.6\,\text{kg})(7.3\,\text{m/s}) - (6.5\,\text{kg})(11.4\,\text{m/s})]/(3.1\,\text{kg}) = -1.30\,\text{m/s}.$$

and

$$v_{2,y} = [(9.6\,\text{kg})(-3.9\,\text{m/s}) - (6.5\,\text{kg})(-4.6\,\text{m/s})]/(3.1\,\text{kg}) = -2.4\,\text{m/s}.$$

$\boxed{\textbf{E7-13}}$ The center of mass should lie on the perpendicular bisector of the rod of mass $3M$. We can view the system as having two parts: the heavy rod of mass $3M$ and the two light rods each of mass $M$. The two light rods have a center of mass at the center of the square.

Both of these center of masses are located along the vertical line of symmetry for the object. The center of mass of the heavy bar is at $y_{\mathrm{h,cm}} = 0$, while the *combined* center of mass of the two light bars is at $y_{\mathrm{l,cm}} = L/2$, where down is positive. The center of mass of the system is then at

$$y_{\mathrm{cm}} = \frac{2My_{\mathrm{l,cm}} + 3My_{\mathrm{h,cm}}}{2M + 3M} = \frac{2(L/2)}{5} = L/5.$$

**E7-14** The two slabs have the same volume and have mass $m_i = \rho_i V$. The center of mass is located at

$$x_{\mathrm{cm}} = \frac{m_1 l - m_2 l}{m_1 + m_2} = l\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2} = (5.5\text{ cm})\frac{(7.85\text{ g/cm}^3) - (2.70\text{ g/cm}^3)}{(7.85\text{ g/cm}^3) + (2.70\text{ g/cm}^3)} = 2.68\text{ cm}$$

from the boundary inside the iron; it is centered in the $y$ and $z$ directions.

**E7-15** Treat the four sides of the box as one thing of mass $4m$ with a mass located $l/2$ above the base. Then the center of mass is

$$z_{\mathrm{cm}} = (l/2)(4m)/(4m + m) = 2l/5 = 2(0.4\,\mathrm{m})/5 = 0.16\,\mathrm{m},$$

$x_{\mathrm{cm}} = y_{\mathrm{cm}} = 0.2\,\mathrm{m}$.

**E7-16** One piece moves off with momentum $m(31.4\,\mathrm{m/s})\hat{\mathbf{i}}$, another moves off with momentum $2m(31.4\,\mathrm{m/s})\hat{\mathbf{j}}$. The third piece must then have momentum $-m(31.4\,\mathrm{m/s})\hat{\mathbf{i}} - 2m(31.4\,\mathrm{m/s})\hat{\mathbf{j}}$ and velocity $-(1/3)(31.4\,\mathrm{m/s})\hat{\mathbf{i}} - 2/3(31.4\,\mathrm{m/s})\hat{\mathbf{j}} = -10.5\,\mathrm{m/s}\,\hat{\mathbf{i}} - 20.9\,\mathrm{m/s}\,\hat{\mathbf{j}}$. The magnitude of $v_3$ is $23.4\,\mathrm{m/s}$ and direction $63.3^\circ$ away from the lighter piece.

**E7-17** It will take an impulse of $(84.7\,\mathrm{kg})(3.87\,\mathrm{m/s}) = 328\,\mathrm{kg\cdot m/s}$ to stop the animal. This would come from firing $n$ bullets where $n = (328\,\mathrm{kg\cdot m/s})/[(0.0126\,\mathrm{kg})(975\,\mathrm{m/s}) = 27$.

**E7-18** Conservation of momentum for firing one cannon ball of mass $m$ with muzzle speed $v_c$ forward out of a cannon on a trolley of original total mass $M$ moving forward with original speed $v_0$ is

$$Mv_0 = (M - m)v_1 + m(v_c + v_1) = Mv_1 + mv_c,$$

where $v_1$ is the speed of the trolley after the cannonball is fired. Then to stop the trolley we require $n$ cannonballs be fired so that

$$n = (Mv_0)/(mv_c) = [(3500\,\mathrm{kg})(45\,\mathrm{m/s})]/[(65\,\mathrm{kg})(625\,\mathrm{m/s})] = 3.88,$$

so $n = 4$.

$\boxed{\textbf{E7-19}}$ Label the velocities of the various containers as $\vec{\mathbf{v}}_k$ where $k$ is an integer between one and twelve. The mass of each container is $m$. The subscript "g" refers to the goo; the subscript $k$ refers to the $k$th container.

The total momentum before the collision is given by

$$\vec{\mathbf{P}} = \sum_k m\vec{\mathbf{v}}_{k,\mathrm{i}} + m_{\mathrm{g}}\vec{\mathbf{v}}_{\mathrm{g,i}} = 12m\vec{\mathbf{v}}_{\mathrm{cont.,cm}} + m_g\vec{\mathbf{v}}_{\mathrm{g,i}}.$$

We are told, however, that the initial velocity of the center of mass of the containers is at rest, so the initial momentum simplifies to $\vec{\mathbf{P}} = m_g \vec{\mathbf{v}}_{\mathrm{g,i}}$, and has a magnitude of 4000 kg·m/s.

(a) Then

$$v_{\mathrm{cm}} = \frac{P}{12m + m_{\mathrm{g}}} = \frac{(4000\,\mathrm{kg\cdot m/s})}{12(100.0\,\mathrm{kg}) + (50\,\mathrm{kg})} = 3.2\,\mathrm{m/s}.$$

(b) It doesn't matter if the cord breaks, we'll get the same answer for the motion of the center of mass.

**E7-20** (a) $F = (3270\,\mathrm{m/s})(480\,\mathrm{kg/s}) = 1.57\times10^{6}\mathrm{N}$.

(b) $m = 2.55\times10^{5}\mathrm{kg} - (480\,\mathrm{kg}/s)(250\,\mathrm{s}) = 1.35\times10^{5}\mathrm{kg}$.

(c) Eq. 7-32:

$$v_{\mathrm{f}} = (-3270\,\mathrm{m/s})\ln(1.35\times10^{5}\mathrm{kg}/2.55\times10^{5}\mathrm{kg}) = 2080\,\mathrm{m/s}.$$

$\boxed{\textbf{E7-21}}$ Use Eq. 7-32. The initial velocity of the rocket is 0. The mass ratio can then be found from a minor rearrangement;

$$\frac{M_{\mathrm{i}}}{M_{\mathrm{f}}} = e^{|v_{\mathrm{f}}/v_{\mathrm{rel}}|}$$

The "flipping" of the left hand side of this expression is possible because the exhaust velocity is *negative* with respect to the rocket. For part (a) $M_{\mathrm{i}}/M_{\mathrm{f}} = e = 2.72$. For part (b) $M_{\mathrm{i}}/M_{\mathrm{f}} = e^2 = 7.39$.

**E7-22** Eq. 7-32 rearranged:

$$\frac{M_{\mathrm{f}}}{M_{\mathrm{i}}} = e^{-|\Delta v/v_{\mathrm{rel}}|} = e^{-(22.6\mathrm{m/s})/(1230\mathrm{m/s})} = 0.982.$$

The fraction of the initial mass which is discarded is 0.0182.

**E7-23** The loaded rocket has a weight of $(1.11\times10^{5}\mathrm{kg})(9.81\,\mathrm{m/s^2}) = 1.09\times10^{6}\mathrm{N}$; the thrust must be at least this large to get the rocket off the ground. Then $v \geq (1.09\times10^{6}\mathrm{N})/(820\,\mathrm{kg/s}) = 1.33\times10^{3}\mathrm{m/s}$ is the minimum exhaust speed.

**E7-24** The acceleration down the incline is $(9.8\,\mathrm{m/s^2})\sin(26^\circ) = 4.3\,\mathrm{m/s^2}$. It will take $t = \sqrt{2(93\,\mathrm{m})/(4.3\,\mathrm{m/s^2})} = 6.6\,\mathrm{s}$. The sand doesn't affect the problem, so long as it only "leaks" out.

$\boxed{\textbf{E7-25}}$ We'll use Eq. 7-4 to solve this problem, but since we are given *weights* instead of *mass* we'll multiply the top and bottom by $g$ like we did in Exercise 7-7. Then

$$\vec{\mathbf{v}}_{\mathrm{cm}} = \frac{m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2}{m_1 + m_2}\frac{g}{g} = \frac{W_1\vec{\mathbf{v}}_1 + W_2\vec{\mathbf{v}}_2}{W_1 + W_2}.$$

Now for the numbers

$$v_{\mathrm{cm}} = \frac{(9.75\ \mathrm{T})(1.36\,\mathrm{m/s}) + (0.50\ \mathrm{T})(0)}{(9.75\ \mathrm{T}) + (0.50\ \mathrm{T})} = 1.29\,\mathrm{m/s}.$$

**P7-1** (a) The balloon moves down so that the center of mass is stationary;

$$0 = Mv_{\mathrm{b}} + mv_{\mathrm{m}} = Mv_{\mathrm{b}} + m(v + v_{\mathrm{b}}),$$

or $v_{\mathrm{b}} = -mv/(m + M)$.

(b) When the man stops so does the balloon.

**P7-2** (a) The center of mass is midway between them.

(b) Measure from the heavier mass.

$$x_{\text{cm}} = (0.0560\,\text{m})(0.816\,\text{kg})/(1.700\,\text{kg}) = 0.0269\,\text{m},$$

which is 1.12 mm closer to the heavier mass than in part (a).

(c) Think Atwood's machine. The acceleration of the two masses is

$$a = 2\Delta m\, g/(m_1 + m_2) = 2(0.034\,\text{kg})g/(1.700\,\text{kg}) = 0.0400g,$$

the heavier going down while the lighter moves up. The acceleration of the center of mass is

$$a_{\text{cm}} = (am_1 - am_2)/(m_1 + m_2) = (0.0400g)2(0.034\,\text{kg})g/(1.700\,\text{kg}) = 0.00160g.$$

$\boxed{\textbf{P7-3}}$ This is a glorified Atwood's machine problem. The total mass on the right side is the mass per unit length times the length, $m_r = \lambda x$; similarly the mass on the left is given by $m_l = \lambda(L - x)$. Then

$$a = \frac{m_2 - m_1}{m_2 + m_1} g = \frac{\lambda x - \lambda(L - x)}{\lambda x + \lambda(L - x)} g = \frac{2x - L}{L} g$$

which solves the problem. The acceleration is in the direction of the side of length $x$ if $x > L/2$.

**P7-4** (a) Assume the car is massless. Then moving the cannonballs is moving the center of mass, unless the cannonballs don't move but instead the car does. How far? $L$.

(b) Once the cannonballs stop moving so does the rail car.

$\boxed{\textbf{P7-5}}$ By symmetry, the center of mass of the empty storage tank should be in the very center, along the axis at a height $y_{\text{t,cm}} = H/2$. We can pretend that the entire mass of the tank, $m_{\text{t}} = M$, is located at this point.

The center of mass of the gasoline is also, by symmetry, located along the axis at half the height of the gasoline, $y_{\text{g,cm}} = x/2$. The mass, if the tank were filled to a height $H$, is $m$; assuming a uniform density for the gasoline, the mass present when the level of gas reaches a height $x$ is $m_g = mx/H$.

(a) The center of mass of the entire system is at the center of the cylinder when the tank is full and when the tank is empty. When the tank is half full the center of mass is below the center. So as the tank changes from full to empty the center of mass drops, reaches some lowest point, and then rises back to the center of the tank.

(b) The center of mass of the entire system is found from

$$y_{\text{cm}} = \frac{m_{\text{g}} y_{\text{g,cm}} + m_{\text{t}} y_{\text{t,cm}}}{m_{\text{g}} + m_{\text{t}}} = \frac{(mx/H)(x/2) + (M)(H/2)}{(mx/H) + (M)} = \frac{mx^2 + MH^2}{2mx + 2MH}.$$

Take the derivative:

$$\frac{dy_{\text{cm}}}{dx} = \frac{m\left(mx^2 + 2xMH - MH^2\right)}{(mx + MH)^2}$$

Set this equal to zero to find the minimum; this means we want the numerator to vanish, or $mx^2 + 2xMH - MH^2 = 0$. Then

$$x = \frac{-M + \sqrt{M^2 + mM}}{m} H.$$

**P7-6** The center of mass will be located along symmetry axis. Call this the $x$ axis. Then

$$
\begin{aligned}
x_{\rm cm} &= \frac{1}{M}\int x dm, \\
&= \frac{4}{\pi R^2}\int_0^R \int_0^{\sqrt{R^2-x^2}} x\,dy\,dx, \\
&= \frac{4}{\pi R^2}\int_0^R x\sqrt{R^2-x^2}\,dx, \\
&= \frac{4}{\pi R^2}R^3/3 = \frac{4R}{3\pi}.
\end{aligned}
$$

**P7-7** (a) The components of the shell velocity with respect to the cannon are

$$v'_x = (556\,\mathrm{m/s})\cos(39.0^\circ) = 432\,\mathrm{m/s} \text{ and } v'_y = (556\,\mathrm{m/s})\sin(39.0^\circ) = 350\,\mathrm{m/s}.$$

The vertical component with respect to the ground is the same, $v_y = v'_y$, but the horizontal component is found from conservation of momentum:

$$M(v_x - v'_x) + m(v_x) = 0,$$

so $v_x = (1400\,\mathrm{kg})(432\,\mathrm{m/s})(70.0\,\mathrm{kg} + 1400\,\mathrm{kg}) = 411\,\mathrm{m/s}$. The resulting speed is $v = 540\,\mathrm{m/s}$.

(b) The direction is $\theta = \arctan(350/411) = 40.4^\circ$.

**P7-8** $v = (2870\,\mathrm{kg})(252\,\mathrm{m/s})/(2870\,\mathrm{kg} + 917\,\mathrm{kg}) = 191\,\mathrm{m/s}$.

**P7-9** It takes $(1.5\,\mathrm{m/s})(20\,\mathrm{kg}) = 30\,\mathrm{N}$ to accelerate the luggage to the speed of the belt. The people when taking the luggage off will (on average) also need to exert a 30 N force to remove it; this force (because of friction) will be exerted on the belt. So the belt requires 60 N of additional force.

**P7-10** (a) The thrust must be at least equal to the weight, so

$$dm/dt = (5860\,\mathrm{kg})(9.81\,\mathrm{m/s^2})/(1170\,\mathrm{m/s}) = 49.1\,\mathrm{kg/s}.$$

(b) The net force on the rocket will need to be $F = (5860\,\mathrm{kg})(18.3\,\mathrm{m/s^2}) = 107000\,\mathrm{N}$. Add this to the weight to find the thrust, so

$$dm/dt = [107000\,\mathrm{N} + (5860\,\mathrm{kg})(9.81\,\mathrm{m/s^2})]/(1170\,\mathrm{m/s}) = 141\,\mathrm{kg/s}$$

$\boxed{\textbf{P7-11}}$ Consider Eq. 7-31. We want the barges to continue at constant speed, so the left hand side of that equation vanishes. Then

$$\sum \vec{\mathbf{F}}_{\rm ext} = -\vec{\mathbf{v}}_{\rm rel}\frac{dM}{dt}.$$

We are told that the frictional force is independent of the weight, since the speed doesn't change the frictional force should be constant and equal in magnitude to the force exerted by the engine *before* the shoveling happens. Then $\sum \vec{\mathbf{F}}_{\rm ext}$ is equal to the additional force required from the engines. We'll call it $\vec{\mathbf{P}}$.

The relative speed of the coal to the faster moving cart has magnitude: $21.2 - 9.65 = 11.6$ km/h= 3.22 m/s. The mass flux is 15.4 kg/s, so $P$ = (3.22 m/s)(15.4kg/s) = 49.6 N. The faster moving cart will need to *increase* the engine force by 49.6 N. The slower cart won't need to do anything, because the coal left the slower barge with a relative speed of *zero* according to our approximation.

**P7-12** (a) Nothing is ejected from the string, so $v_{\mathrm{rel}} = 0$. Then Eq. 7-31 reduces to $m\,dv/dt = F_{\mathrm{ext}}$.

(b) Since $F_{\mathrm{ext}}$ is from the weight of the hanging string, and the fraction that is hanging is $y/L$, $F_{\mathrm{ext}} = mgy/L$. The equation of motion is then $d^2y/dt^2 = gy/L$.

(c) Take first derivative:

$$\frac{dy}{dt} = \frac{y_0}{2}(\sqrt{g/L})\left(e^{\sqrt{g/L}t} - e^{-\sqrt{g/L}t}\right),$$

and then second derivative,

$$\frac{d^2y}{dt^2} = \frac{y_0}{2}(\sqrt{g/L})^2\left(e^{\sqrt{g/L}t} + e^{-\sqrt{g/L}t}\right).$$

Substitute into equation of motion. It works! Note that when $t = 0$ we have $y = y_0$.