E16-1 $R = Av = \pi d^2 v/4$ and $V = Rt$, so

$$t = \frac{4(1600\,\mathrm{m}^3)}{\pi(0.345\,\mathrm{m})^2(2.62\,\mathrm{m})} = 6530\,\mathrm{s}$$

E16-2 $A_1 v_1 = A_2 v_2$, $A_1 = \pi d_1^2/4$ for the hose, and $A_2 = N\pi d_2^2$ for the sprinkler, where $N = 24$. Then

$$v_2 = \frac{(0.75\text{ in})^2}{(24)(0.050\text{ in})^2}(3.5\text{ ft/s}) = 33\text{ ft/s}.$$

$\boxed{\textbf{E16-3}}$ We'll assume that each river has a rectangular cross section, despite what the picture implies. The cross section area of the two streams is then

$$A_1 = (8.2\,\mathrm{m})(3.4\,\mathrm{m}) = 28\,\mathrm{m}^2 \text{ and } A_2 = (6.8\,\mathrm{m})(3.2\,\mathrm{m}) = 22\,\mathrm{m}^2.$$

The volume flow rate in the first stream is

$$R_1 = A_1 v_1 = (28\,\mathrm{m}^2)(2.3\,\mathrm{m/s}) = 64\,\mathrm{m}^3/\mathrm{s},$$

while the volume flow rate in the second stream is

$$R_2 = A_2 v_2 = (22\,\mathrm{m}^2)(2.6\,\mathrm{m/s}) = 57\,\mathrm{m}^3/\mathrm{s}.$$

The amount of fluid in the stream/river system is conserved, so

$$R_3 = R_1 + R_2 = (64\,\mathrm{m}^3/\mathrm{s}) + (57\,\mathrm{m}^3/\mathrm{s}) = 121\,\mathrm{m}^3/\mathrm{s}.$$

where R_3 is the volume flow rate in the river. Then

$$D_3 = R_3/(v_3 W_3) = (121\,\mathrm{m}^3/\mathrm{s})/[(10.7\,\mathrm{m})(2.9\,\mathrm{m/s})] = 3.9\,\mathrm{m}.$$

E16-4 The speed of the water is originally zero so both the kinetic and potential energy is zero. When it leaves the pipe at the top it has a kinetic energy of $\frac{1}{2}(5.30\,\mathrm{m/s})^2 = 14.0\,\mathrm{J/kg}$ and a potential energy of $(9.81\,\mathrm{m/s}^2)(2.90\,\mathrm{m}) = 28.4\,\mathrm{J/kg}$. The water is flowing out at a volume rate of $R = (5.30\,\mathrm{m/s})\pi(9.70\times10^{-3}\mathrm{m})^2 = 1.57\times10^{-3}\,\mathrm{m}^3/\mathrm{s}$. The mass rate is $\rho R = (1000\,\mathrm{kg/m}^3)(1.57\times10^{-3}\,\mathrm{m}^3/\mathrm{s}) = 1.57\,\mathrm{kg/s}$.

The power supplied by the pump is $(42.8\,\mathrm{J/kg})(1.57\,\mathrm{kg/s}) = 67.2\mathrm{W}$.

$\boxed{\textbf{E16-5}}$ There are 8500 km^2 which collects an average of (0.75)(0.48 m/y), where the 0.75 reflects the fact that 1/4 of the water evaporates, so

$$R = \left[8500(10^3\,\mathrm{m})^2\right](0.75)(0.48\,\mathrm{m/y})\left(\frac{1\text{ y}}{365\times24\times60\times60\,\mathrm{s}}\right) = 97\,\mathrm{m}^3/\mathrm{s}.$$

Then the speed of the water in the river is

$$v = R/A = (97\,\mathrm{m}^3/\mathrm{s})/[(21\,\mathrm{m})(4.3\,\mathrm{m})] = 1.1\,\mathrm{m/s}.$$

E16-7 (a) $\Delta p = \rho g(y_1 - y_2) + \rho(v_1^2 - v_2^2)/2$. Then

$$\Delta p = (62.4\text{ lb/ft}^3)(572\text{ ft}) + [(62.4\text{ lb/ft}^3)/(32\text{ ft/s}^2)][(1.33\text{ ft/s})^2 - (31.0\text{ ft/s})^2]/2 = 3.48\times10^4\text{lb/ft}^2.$$

(b) $A_2 v_2 = A_1 v_1$, so

$$A_2 = (7.60\text{ ft}^2)(1.33\text{ ft/s})/(31.0\text{ ft/s}) = 0.326\text{ ft}^2.$$

E16-8 (a) $A_2 v_2 = A_1 v_1$, so

$$v_2 = (2.76\,\text{m/s})[(0.255\,\text{m})^2 - (0.0480\,\text{m})^2]/(0.255\,\text{m})^2 = 2.66\,\text{m/s}.$$

(b) $\Delta p = \rho(v_1^2 - v_2^2)/2$,

$$\Delta p = (1000\,\text{kg/m}^3)[(2.66\,\text{m/s})^2 - (2.76\,\text{m/s})^2]/2 = -271\,\text{Pa}$$

$\boxed{\textbf{E16-9}}$ (b) We will do part (b) first.

$$R = (100\,\text{m}^2)(1.6\,\text{m/y})\left(\frac{1\text{ y}}{365 \times 24 \times 60 \times 60\,\text{s}}\right) = 5.1\times10^{-6}\,\text{m}^3/\text{s}.$$

(b) The speed of the flow R through a hole of cross sectional area a will be $v = R/a$. $p = p_0 + \rho g h$, where $h = 2.0\,\text{m}$ is the depth of the hole. Bernoulli's equation can be applied to find the speed of the water as it travels a horizontal stream line out the hole,

$$p_0 + \frac{1}{2}\rho v^2 = p,$$

where we drop any terms which are either zero or the same on both sides. Then

$$v = \sqrt{2(p - p_0)/\rho} = \sqrt{2gh} = \sqrt{2(9.81\,\text{m/s}^2)(2.0\,\text{m})} = 6.3\,\text{m/s}.$$

Finally, $a = (5.1\times10^{-6}\,\text{m}^3/\text{s})/(6.3\,\text{m/s}) = 8.1\times10^{-7}\text{m}^2$, or about 0.81 mm^2.

E16-10 (a) $v_2 = (A_1/A_2)v_1 = (4.20\text{ cm}^2)(5.18\,\text{m/s})/(7.60\text{ cm}^2) = 2.86\,\text{m/s}$.
(b) Use Bernoulli's equation:

$$p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2 = p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2.$$

Then

$$\begin{aligned} p_2 &= (1.52\times10^5\text{Pa}) + (1000\,\text{kg/m}^3)\left[(9.81\,\text{m/s}^2)(9.66\,\text{m}) + \frac{1}{2}(5.18\,\text{m/s})^2 - \frac{1}{2}(2.86\,\text{m/s})^2\right], \\ &= 2.56\times10^5\text{Pa}. \end{aligned}$$

E16-11 (a) The wind speed is (110 km/h)(1000 m/km)/(3600 s/h) = $30.6\,\text{m/s}$. The pressure difference is then

$$\Delta p = \frac{1}{2}(1.2\,\text{kg/m}^3)(30.6\,\text{m/s})^2 = 562\,\text{Pa}.$$

(b) The lifting force would be $F = (562\,\text{Pa})(93\,\text{m}^2) = 52000\,\text{N}$.

E16-12 The pressure difference is

$$\Delta p = \frac{1}{2}(1.23\,\text{kg/m}^3)(28.0\,\text{m/s})^2 = 482\,\text{N}.$$

The net force is then $F = (482\,\text{N})(4.26\,\text{m})(5.26\,\text{m}) = 10800\,\text{N}$.

$\boxed{\textbf{E16-13}}$ The lower pipe has a radius $r_1 = 2.52$ cm, and a cross sectional area of $A_1 = \pi r_1^2$. The speed of the fluid flow at this point is v_1. The higher pipe has a radius of $r_2 = 6.14$ cm, a cross sectional area of $A_2 = \pi r_2^2$, and a fluid speed of v_2. Then

$$A_1 v_1 = A_2 v_2 \text{ or } r_1^2 v_1 = r_2^2 v_2.$$

Set $y_1 = 0$ for the lower pipe. The problem specifies that the pressures in the two pipes are the same, so

$$\begin{aligned} p_0 + \frac{1}{2}\rho v_1^2 + \rho g y_1 &= p_0 + \frac{1}{2}\rho v_2^2 + \rho g y_2, \\ \frac{1}{2} v_1^2 &= \frac{1}{2} v_2^2 + g y_2, \end{aligned}$$

We can combine the results of the equation of continuity with this and get

$$\begin{aligned} v_1^2 &= v_2^2 + 2 g y_2, \\ v_1^2 &= \left(v_1 r_1^2 / r_2^2\right)^2 + 2 g y_2, \\ v_1^2 \left(1 - r_1^4/r_2^4\right) &= 2 g y_2, \\ v_1^2 &= 2 g y_2 / \left(1 - r_1^4/r_2^4\right). \end{aligned}$$

Then

$$v_1^2 = 2(9.81\,\mathrm{m/s^2})(11.5\,\mathrm{m}) / \left(1 - (0.0252\,\mathrm{m})^4/(0.0614\,\mathrm{m})^4\right) = 232\,\mathrm{m^2/s^2}$$

The volume flow rate in the bottom (and top) pipe is

$$R = \pi r_1^2 v_1 = \pi (0.0252\,\mathrm{m})^2 (15.2\,\mathrm{m/s}) = 0.0303\,\mathrm{m^3/s}.$$

E16-14 (a) As instructed,

$$\begin{aligned} p_0 + \frac{1}{2}\rho v_1^2 + \rho g y_1 &= p_0 + \frac{1}{2}\rho v_3^2 + \rho g y_3, \\ 0 &= \frac{1}{2} v_3^2 + g(y_3 - y_1), \end{aligned}$$

But $y_3 - y_1 = -h$, so $v_3 = \sqrt{2gh}$.

(b) h above the hole. Just reverse your streamline!

(c) It won't come out as fast and it won't rise as high.

$\boxed{\textbf{E16-15}}$ Sea level will be defined as $y = 0$, and at that point the fluid is assumed to be at rest. Then

$$\begin{aligned} p_0 + \frac{1}{2}\rho v_1^2 + \rho g y_1 &= p_0 + \frac{1}{2}\rho v_2^2 + \rho g y_2, \\ 0 &= \frac{1}{2} v_2^2 + g y_2, \end{aligned}$$

where $y_2 = -200$ m. Then

$$v_2 = \sqrt{-2 g y_2} = \sqrt{-2(9.81\,\mathrm{m/s^2})(-200\,\mathrm{m})} = 63\,\mathrm{m/s}.$$

E16-16 Assume streamlined flow, then

$$\begin{aligned} p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 &= p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2, \\ (p_1 - p_2)/\rho + g(y_1 - y_2) &= \frac{1}{2}v_2^2. \end{aligned}$$

Then upon rearranging

$$v_2 = \sqrt{2\left[(2.1)(1.01\times10^5\text{Pa})/(1000\,\text{kg/m}^3) + (9.81\,\text{m/s}^2)(53.0\,\text{m})\right]} = 38.3\,\text{m/s}.$$

E16-17 (a) Points 1 and 3 are both at atmospheric pressure, and both will move at the same speed. But since they are at different heights, Bernoulli's equation will be violated.

(b) The flow isn't steady.

E16-18 The atmospheric pressure difference between the two sides will be $\Delta p = \frac{1}{2}\rho_\text{a} v^2$. The height difference in the U-tube is given by $\Delta p = \rho_\text{w} g h$. Then

$$h = \frac{(1.20\,\text{kg/m}^3)(15.0\,\text{m/s})^2}{2(1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)} = 1.38\times10^{-2}\text{m}.$$

$\boxed{\textbf{E16-19}}$ (a) There are three forces on the plug. The force from the pressure of the water, $F_1 = P_1 A$, the force from the pressure of the air, $F_2 = P_2 A$, and the force of friction, F_3. These three forces must balance, so $F_3 = F_1 - F_2$, or $F_3 = P_1 A - P_2 A$. But $P_1 - P_2$ is the pressure difference between the surface and the water 6.15 m below the surface, so

$$\begin{aligned} F_3 &= \Delta P A = -\rho g y A, \\ &= -(998\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(-6.15\,\text{m})\pi(0.0215\,\text{m})^2, \\ &= 87.4\,\text{N} \end{aligned}$$

(b) To find the volume of water which flows out in three hours we need to know the volume flow rate, and for that we need both the cross section area of the hole and the speed of the flow. The speed of the flow can be found by an application of Bernoulli's equation. We'll consider the horizontal motion only— a point just inside the hole, and a point just outside the hole. These points are at the same level, so

$$\begin{aligned} p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 &= p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2, \\ p_1 &= p_2 + \frac{1}{2}\rho v_2^2. \end{aligned}$$

Combine this with the results of Pascal's principle above, and

$$v_2 = \sqrt{2(p_1 - p_2)/\rho} = \sqrt{-2gy} = \sqrt{-2(9.81\,\text{m/s}^2)(-6.15\,\text{m})} = 11.0\,\text{m/s}.$$

The volume of water which flows out in three hours is

$$V = Rt = (11.0\,\text{m/s})\pi(0.0215\,\text{m})^2(3\times3600\,\text{s}) = 173\,\text{m}^3.$$

E16-20 Apply Eq. 16-12:

$$v_1 = \sqrt{2(9.81\,\text{m/s}^2)(0.262\,\text{m})(810\,\text{kg/m}^3)/(1.03\,\text{kg/m}^3)} = 63.6\,\text{m/s}.$$

$\boxed{\textbf{E16-21}}$ We'll assume that the central column of air down the pipe exerts minimal force on the card when it is deflected to the sides. Then

$$\begin{aligned} p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 &= p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2, \\ p_1 &= p_2 + \frac{1}{2}\rho v_2^2. \end{aligned}$$

The resultant upward force on the card is the area of the card times the pressure difference, or

$$F = (p_1 - p_2)A = \frac{1}{2}\rho A v^2.$$

E16-22 If the air blows uniformly over the surface of the plate then there can be no torque about any axis through the center of mass of the plate. Since the weight also doesn't introduce a torque, then the hinge can't exert a force on the plate, because any such force would produce an unbalanced torque. Consequently $mg = \Delta p A$. $\Delta p = \rho v^2/2$, so

$$v = \sqrt{\frac{2mg}{\rho A}} = \sqrt{\frac{2(0.488\,\mathrm{kg})(9.81\,\mathrm{m/s^2})}{(1.21\,\mathrm{kg/m^3})(9.10\times10^{-2}\mathrm{m})^2}} = 30.9\,\mathrm{m/s}.$$

E16-23 Consider a streamline which passes above the wing and a streamline which passes beneath the wing. Far from the wing the two streamlines are close together, move with zero relative velocity, and are effectively at the same pressure. So we can pretend they are actually one streamline. Then, since the altitude difference between the two points above and below the wing (on this new, single streamline) is so small we can write

$$\Delta p = \frac{1}{2}\rho({v_\mathrm{t}}^2 - {v_\mathrm{u}}^2)$$

The lift force is then

$$L = \Delta p\, A = \frac{1}{2}\rho A({v_\mathrm{t}}^2 - {v_\mathrm{u}}^2)$$

E16-24 (a) From Exercise 16-23,

$$L = \frac{1}{2}(1.17\,\mathrm{kg/m^3})(2)(12.5\,\mathrm{m^2})\left[(49.8\,\mathrm{m/s})^2 - (38.2\,\mathrm{m/s})^2\right] = 1.49\times10^4\mathrm{N}.$$

The mass of the plane must be $m = L/g = (1.49\times10^4\mathrm{N})/(9.81\,\mathrm{m/s^2}) = 1520\,\mathrm{kg}$.

(b) The lift is directed straight up.

(c) The lift is directed 15° off the vertical toward the rear of the plane.

(d) The lift is directed 15° off the vertical toward the front of the plane.

$\boxed{\textbf{E16-25}}$ The larger pipe has a radius $r_1 = 12.7$ cm, and a cross sectional area of $A_1 = \pi r_1^2$. The speed of the fluid flow at this point is v_1. The smaller pipe has a radius of $r_2 = 5.65$ cm, a cross sectional area of $A_2 = \pi r_2^2$, and a fluid speed of v_2. Then

$$A_1 v_1 = A_2 v_2 \text{ or } r_1^2 v_1 = r_2^2 v_2.$$

Now Bernoulli's equation. The two pipes are at the same level, so $y_1 = y_2$. Then

$$\begin{aligned} p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 &= p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2, \\ p_1 + \frac{1}{2}\rho v_1^2 &= p_2 + \frac{1}{2}\rho v_2^2. \end{aligned}$$

Combining this with the results from the equation of continuity,

$$
\begin{aligned}
p_1 + \frac{1}{2}\rho v_1^2 &= p_2 + \frac{1}{2}\rho v_2^2, \\
v_1^2 &= v_2^2 + \frac{2}{\rho}\left(p_2 - p_1\right), \\
v_1^2 &= \left(v_1 \frac{r_1^2}{r_2^2}\right)^2 + \frac{2}{\rho}\left(p_2 - p_1\right), \\
v_1^2\left(1 - \frac{r_1^4}{r_2^4}\right) &= \frac{2}{\rho}\left(p_2 - p_1\right), \\
v_1^2 &= \frac{2\left(p_2 - p_1\right)}{\rho\left(1 - r_1^4/r_2^4\right)}.
\end{aligned}
$$

It may look a mess, but we can solve it to find v_1,

$$
v_1 = \sqrt{\frac{2(32.6\times10^3\text{Pa} - 57.1\times10^3\text{Pa})}{(998\,\text{kg/m}^3)(1-(0.127\,\text{m})^4/(0.0565\,\text{m})^4)}} = 1.41\,\text{m/s}.
$$

The volume flow rate is then

$$
R = Av = \pi(0.127\,\text{m})^2(1.41\,\text{m/s}) = 7.14\times10^{-3}\text{m}^3/\text{s}.
$$

That's about 71 liters/second.

E16-26 The lines are parallel and equally spaced, so the velocity is everywhere the same. We can transform to a reference frame where the liquid appears to be at rest, so the Pascal's equation would apply, and $p + \rho g y$ would be a constant. Hence,

$$
p_0 = p + \rho g y + \frac{1}{2}\rho v^2
$$

is the same for all streamlines.

E16-27 (a) The "particles" of fluid in a whirlpool would obey conservation of angular momentum, meaning a particle off mass m would have $l = mvr$ be constant, so the speed of the fluid as a function of radial distance from the center would be given by $k = vr$, where k is some constant representing the angular momentum per mass. Then $v = k/r$.

(b) Since $v = k/r$ and $v = 2\pi r/T$, the period would be $T \propto r^2$.

(c) Kepler's third law says $T \propto r^{3/2}$.

E16-28 $R_\text{c} = 2000$. Then

$$
v < \frac{R_\text{c}\eta}{\rho D} = \frac{(2000)(4.0\times10^{-3}\text{N}\cdot\text{s/m}^2)}{(1060\,\text{kg/m}^3)(3.8\times10^{-3}\text{m})} = 2.0\,\text{m/s}.
$$

$\boxed{\textbf{E16-29}}$ (a) The volume flux is given; from that we can find the average speed of the fluid in the pipe.

$$
v = \frac{5.35\times10^{-2}\ \text{L/min}}{\pi(1.88\ \text{cm})^2} = 4.81\times10^{-3}\ \text{L/cm}^2\cdot\text{min}.
$$

But 1 L is the same as 1000 cm^3 and 1 min is equal to 60 seconds, so $v = 8.03\times10^{-4}$ m/s.

Reynold's number from Eq. 16-22 is then

$$R = \frac{\rho D v}{\eta} = \frac{(13600\,\text{kg/m}^3)(0.0376\,\text{m})(8.03\times10^{-4}\,\text{m/s})}{(1.55\times10^{-3}\text{N}\cdot\text{s/m}^2)} = 265.$$

This is well below the critical value of 2000.

(b) Poiseuille's Law, Eq. 16-20, can be used to find the pressure difference between the ends of the pipe. But first, note that the mass flux dm/dt is equal to the volume rate times the density when the density is constant. Then $\rho\, dV/dt = dm/dt$, and Poiseuille's Law can be written as

$$\delta p = \frac{8\eta L}{\pi R^4}\frac{dV}{dt} = \frac{8(1.55\times10^{-3}\text{N}\cdot\text{s/m}^2)(1.26\,\text{m})}{\pi(1.88\times10^{-2}\ \text{m})^4}(8.92\times10^{-7}\text{m}^3/\text{s}) = 0.0355\,\text{Pa}.$$

P16-1 The volume of water which needs to flow out of the bay is

$$V = (6100\,\text{m})(5200\,\text{m})(3\,\text{m}) = 9.5\times10^7\text{m}^3$$

during a 6.25 hour (22500 s) period. The average speed through the channel must be

$$v = \frac{(9.5\times10^7\text{m}^3)}{(22500\,\text{s})(190\,\text{m})(6.5\,\text{m})} = 3.4\,\text{m/s}.$$

P16-2 (a) The speed of the fluid through either hole is $v = \sqrt{2gh}$. The mass flux through a hole is $Q = \rho A v$, so $\rho_1 A_1 = \rho_2 A_2$. Then $\rho_1/\rho_2 = A_2/A_1 = 2$.

(b) $R = Av$, so $R_1/R_2 = A_1/A_2 = 1/2$.

(c) If $R_1 = R_2$ then $A_1\sqrt{2gh_1} = A_2\sqrt{2gh_2}$. Then

$$h_2/h_1 = (A_1/A_2)^2 = (1/2)^2 = 1/4.$$

So $h_2 = h_1/4$.

$\boxed{\textbf{P16-3}}$ (a) Apply Torricelli's law (Exercise 16-14): $v = \sqrt{2gh}$. The speed v is a horizontal velocity, and serves as the initial horizontal velocity of the fluid "projectile" after it leaves the tank. There is no initial vertical velocity.

This fluid "projectile" falls through a vertical distance $H - h$ before splashing against the ground. The equation governing the time t for it to fall is

$$-(H-h) = -\frac{1}{2}gt^2,$$

Solve this for the time, and $t = \sqrt{2(H-h)/g}$. The equation which governs the horizontal distance traveled during the fall is $x = v_x t$, but $v_x = v$ and we just found t, so

$$x = v_x t = \sqrt{2gh}\,\sqrt{2(H-h)/g} = 2\sqrt{h(H-h)}.$$

(b) How many values of h will lead to a distance of x? We need to invert the expression, and we'll start by squaring both sides

$$x^2 = 4h(H-h) = 4hH - 4h^2,$$

and then solving the resulting quadratic expression for h,

$$h = \frac{4H \pm \sqrt{16H^2 - 16x^2}}{8} = \frac{1}{2}\left(H \pm \sqrt{H^2 - x^2}\right).$$

For values of x between 0 and H there are two real solutions, if $x = H$ there is one real solution, and if $x > H$ there are no real solutions.

If h_1 is a solution, then we can write $h_1 = (H + \Delta)/2$, where $\Delta = 2h_1 - H$ could be positive or negative. Then $h_2 = (H + \Delta)/2$ is also a solution, and

$$h_2 = (H + 2h_1 - 2H)/2 = h_1 - H$$

is also a solution.

(c) The farthest distance is $x = H$, and this happens when $h = H/2$, as we can see from the previous section.

P16-4 (a) Apply Torricelli's law (Exercise 16-14): $v = \sqrt{2g(d + h_2)}$, assuming that the liquid remains in contact with the walls of the tube until it exits at the bottom.

(b) The speed of the fluid in the tube is everywhere the same. Then the pressure difference at various points are only functions of height. The fluid exits at C, and assuming that it remains in contact with the walls of the tube the pressure difference is given by $\Delta p = \rho(h_1 + d + h_2)$, so the pressure at B is

$$p = p_0 - \rho(h_1 + d + h_2).$$

(c) The lowest possible pressure at B is zero. Assume the flow rate is *so* slow that Pascal's principle applies. Then the maximum height is given by $0 = p_0 + \rho g h_1$, or

$$h_1 = (1.01\times 10^5\text{Pa})/[(9.81\,\text{m/s}^2)(1000\,\text{kg/m}^3)] = 10.3\,\text{m}.$$

P16-5 (a) The momentum per kilogram of the fluid in the smaller pipe is v_1. The momentum per kilogram of the fluid in the larger pipe is v_2. The change in momentum per kilogram is $v_2 - v_1$. There are $\rho a_2 v_2$ kilograms per second flowing past any point, so the change in momentum per second is $\rho a_2 v_2(v_2 - v_1)$. The change in momentum per second is related to the net force according to $F = \Delta p/\Delta t$, so $F = \rho a_2 v_2(v_2 - v_1)$. But $F \approx \Delta p/a_2$, so $p_1 - p_2 \approx \rho v_2(v_2 - v_1)$.

(b) Applying the streamline equation,

$$\begin{aligned}
p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 &= p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2, \\
\frac{1}{2}\rho(v_1^2 - v_2^2) &= p_2 - p_1
\end{aligned}$$

(c) This question asks for the loss of pressure *beyond* that which would occur from a gradually widened pipe. Then we want

$$\begin{aligned}
\Delta p &= \frac{1}{2}\rho(v_1^2 - v_2^2) - \rho v_2(v_1 - v_2), \\
&= \frac{1}{2}\rho(v_1^2 - v_2^2) - \rho v_2 v_1 + \rho v_2^2, \\
&= \frac{1}{2}\rho(v_1^2 - 2v_1 v_2 + v_2^2) = \frac{1}{2}\rho(v_1 - v_2)^2.
\end{aligned}$$

P16-6 The juice leaves the jug with a speed $v = \sqrt{2gy}$, where y is the height of the juice in the jug. If A is the cross sectional area of the base of the jug and a the cross sectional area of the hole, then the juice flows out the hole with a rate $dV/dt = va = a\sqrt{2gy}$, which means the level of jug varies as $dy/dt = -(a/A)\sqrt{2gy}$. Rearrange and integrate,

$$\int_y^h dy/\sqrt{y} = \int_0^t \sqrt{2g}(a/A)dt,$$

$$\begin{aligned} 2(\sqrt{h}-\sqrt{y}) &= \sqrt{2g}at/A. \\ \left(\frac{A}{a}\sqrt{\frac{2h}{g}}\right)(\sqrt{h}-\sqrt{y}) &= t \end{aligned}$$

When $y = 14h/15$ we have $t = 12.0\,\mathrm{s}$. Then the part in the parenthesis on the left is $3.539\times10^2\mathrm{s}$. The time to empty completely is then 354 seconds, or 5 minutes and 54 seconds. But we want the remaining time, which is 12 seconds less than this.

P16-7 The greatest possible value for v will be the value over the wing which results in an air pressure of zero. If the air at the leading edge is stagnant (not moving) and has a pressure of p_0, then Bernoulli's equation gives

$$p_0 = \frac{1}{2}\rho v^2,$$

or $v = \sqrt{2p_0/\rho} = \sqrt{2(1.01\times10^5\mathrm{Pa})/(1.2\,\mathrm{kg/m^3})} = 410\,\mathrm{m/s}$. This value is only slightly larger than the speed of sound; they are related because sound waves involve the movement of air particles which "shove" other air particles out of the way.

P16-8 Bernoulli's equation together with continuity gives

$$\begin{aligned} p_1 + \frac{1}{2}\rho v_1^2 &= p_2 + \frac{1}{2}\rho v_2^2, \\ p_1 - p_2 &= \frac{1}{2}\rho\left(v_2^2 - v_1^2\right), \\ &= \frac{1}{2}\rho\left(\frac{A_1^2}{A_2^2}v_1^2 - v_1^2\right), \\ &= \frac{v_1^2}{2A_2^2}\rho\left(A_1^2 - A_2^2\right). \end{aligned}$$

But $p_1 - p_2 = (\rho' - \rho)gh$. Note that we are *not* assuming ρ is negligible compared to ρ'. Combining,

$$v_1 = A_2\sqrt{\frac{2(\rho'-\rho)gh}{\rho(A_1^2 - A_2^2)}}.$$

P16-9 (a) Bernoulli's equation together with continuity gives

$$\begin{aligned} p_1 + \frac{1}{2}\rho v_1^2 &= p_2 + \frac{1}{2}\rho v_2^2, \\ p_1 &= \frac{1}{2}\rho\left(v_2^2 - v_1^2\right), \\ &= \frac{1}{2}\rho\left(\frac{A_1^2}{A_2^2}v_1^2 - v_1^2\right), \\ &= v_1^2\rho\left[(4.75)^2 - 1\right]/2. \end{aligned}$$

Then

$$v_1 = \sqrt{\frac{2(2.12)(1.01\times10^5\mathrm{Pa})}{(1000\,\mathrm{kg/m^3})(21.6)}} = 4.45\,\mathrm{m/s},$$

and then $v_2 = (4.75)(4.45\,\mathrm{m/s}) = 21.2\,\mathrm{m/s}$.

(b) $R = \pi(2.60\times10^{-2}\mathrm{m})^2(4.45\,\mathrm{m/s}) = 9.45\times10^{-3}\,\mathrm{m^3/s}$.

P16-10 (a) For Fig. 16-13 the velocity is constant, or $\vec{\mathbf{v}} = v\hat{\mathbf{i}}$. $d\vec{\mathbf{s}} = \hat{\mathbf{i}}dx + \hat{\mathbf{j}}dy$. Then

$$\oint \vec{\mathbf{v}} \cdot d\vec{\mathbf{s}} = v \oint dx = 0,$$

because $\oint dx = 0$.

(b) For Fig. 16-16 the velocity is $\vec{\mathbf{v}} = (k/r)\hat{\mathbf{r}}$. $d\vec{\mathbf{s}} = \hat{\mathbf{r}}dr + \hat{\theta}r\,d\phi$. Then

$$\oint \vec{\mathbf{v}} \cdot d\vec{\mathbf{s}} = v \oint dr = 0,$$

because $\oint dr = 0$.

P16-11 (a) For an element of the fluid of mass dm the net force as it moves around the circle is $dF = (v^2/r)dm$. $dm/dV = \rho$ and $dV = A\,dr$ and $dF/A = dp$. Then $dp/dr = \rho v^2/r$.

(b) From Bernoulli's equation $p + \rho v^2/2$ is a constant. Then

$$\frac{dp}{dr} + \rho v \frac{dv}{dr} = 0,$$

or $v/r + dv/dr = 0$, or $d(vr) = 0$. Consequently vr is a constant.

(c) The velocity is $\vec{\mathbf{v}} = (k/r)\hat{\mathbf{r}}$. $d\vec{\mathbf{s}} = \hat{\mathbf{r}}dr + \hat{\theta}r\,d\phi$. Then

$$\oint \vec{\mathbf{v}} \cdot d\vec{\mathbf{s}} = v \oint dr = 0,$$

because $\oint dr = 0$. This means the flow is irrotational.

P16-12 $F/A = \eta v/D$, so

$$F/A = (4.0\times10^{19}\mathrm{N\cdot s/m^2})(0.048\,\mathrm{m}/3.16\times10^7\mathrm{s})/(1.9\times10^5\mathrm{m}) = 3.2\times10^5\mathrm{Pa}.$$

$\boxed{\textbf{P16-13}}$ A flow will be irrotational if and only if $\oint \vec{\mathbf{v}} \cdot d\vec{\mathbf{s}} = 0$ for all possible paths. It is fairly easy to construct a rectangular path which is parallel to the flow on the top and bottom sides, but perpendicular on the left and right sides. Then only the top and bottom paths contribute to the integral. $\vec{\mathbf{v}}$ is constant for either path (but not the same), so the magnitude v will come out of the integral sign. Since the lengths of the two paths are the same but v is different the two terms *don't* cancel, so the flow is not irrotational.

P16-14 (a) The area of a cylinder is $A = 2\pi rL$. The velocity gradient if dv/dr. Then the retarding force on the cylinder is $F = -\eta(2\pi rL)dv/dr$.

(b) The force pushing a cylinder through is $F' = A\Delta p = \pi r^2\Delta p$.

(c) Equate, rearrange, and integrate:

$$\begin{aligned}
\pi r^2 \Delta p &= -\eta(2\pi rL)\frac{dv}{dr}, \\
\Delta p \int_r^R r\,dr &= 2\eta L \int_0^v dv, \\
\Delta p \frac{1}{2}(R^2 - r^2) &= 2\eta L v.
\end{aligned}$$

Then

$$v = \frac{\Delta p}{4\eta L}(R^2 - r^2).$$

$\boxed{\textbf{P16-15}}$ The volume flux (called R_f to distinguish it from the radius R) through an annular ring of radius r and width δr is

$$\delta R_f = \delta A\, v = 2\pi r\, \delta r\, v,$$

where v is a function of r given by Eq. 16-18. The mass flux is the volume flux times the density, so the total mass flux is

$$\begin{aligned}
\frac{dm}{dt} &= \rho \int_0^R \frac{\delta R_f}{\delta r} dr, \\
&= \rho \int_0^R 2\pi r \left(\frac{\Delta p}{4\eta L}(R^2 - r^2) \right) dr, \\
&= \frac{\pi \rho \Delta p}{2\eta L} \int_0^R (rR^2 - r^3) dr, \\
&= \frac{\pi \rho \Delta p}{2\eta L} (R^4/2 - R^4/4), \\
&= \frac{\pi \rho \Delta p R^4}{8\eta L}.
\end{aligned}$$

P16-16 The pressure difference in the tube is $\Delta p = 4\gamma/r$, where r is the (changing) radius of the bubble. The mass flux through the tube is

$$\frac{dm}{dt} = \frac{4\rho\pi R^4 \gamma}{8\eta L r},$$

R is the radius of the tube. $dm = \rho dV$, and $dV = 4\pi r^2 dr$. Then

$$\begin{aligned}
\int_{r_1}^{r_2} r^3\, dr &= \int_0^t \frac{R^4 \gamma}{8\eta L} dt, \\
r_1^4 - r_2^4 &= \frac{\rho R^4 \gamma}{2\eta L} t,
\end{aligned}$$

Then

$$t = \frac{2(1.80\times 10^{-5}\mathrm{N\cdot s/m^2})(0.112\,\mathrm{m})}{(0.54\times 10^{-3}\mathrm{m})^4 (2.50\times 10^{-2}\mathrm{N/m})} [(38.2\times 10^{-3}\mathrm{m})^4 - (21.6\times 10^{-3}\mathrm{m})^4] = 3630\,\mathrm{s}.$$