E13-1 If the projectile had not experienced air drag it would have risen to a height \( y_2 \), but because of air drag 68 kJ of mechanical energy was dissipated so it only rose to a height \( y_1 \). In either case the initial velocity, and hence initial kinetic energy, was the same; and the velocity at the highest point was zero. Then \( W = \Delta U \), so the potential energy would have been 68 kJ greater, and
\[
\Delta y = \frac{\Delta U}{mg} = \frac{(68 \times 10^3 \text{J})/(9.4 \text{ kg})(9.81 \text{ m/s}^2)}{9.4 \text{ kg}} = 740 \text{ m}
\]
is how much higher it would have gone without air friction.

E13-2 (a) The road incline is \( \theta = \arctan(0.08) = 4.57^\circ \). The frictional forces are the same; the car is now moving with a vertical upward speed of \( (15 \text{ m/s}) \sin(4.57^\circ) = 1.20 \text{ m/s} \). The additional power required to drive up the hill is then \( \Delta P = m g v_y = (1700 \text{ kg})(9.81 \text{ m/s}^2)(1.20 \text{ m/s}) = 20000 \text{ W} \). The total power required is 36000 W.
(b) The car will “coast” if the power generated by rolling downhill is equal to 16000 W, or
\[
v_y = \frac{(16000 \text{ W})}{[(1700 \text{ kg})(9.81 \text{ m/s}^2)]} = 0.959 \text{ m/s},
\]
down. Then the incline is
\[
\theta = \arcsin(0.959 \text{ m/s}/15 \text{ m/s}) = 3.67^\circ.
\]
This corresponds to a downward grade of \( \tan(3.67^\circ) = 6.4\% \).

E13-3 Apply energy conservation:
\[
\frac{1}{2} m v^2 + m g y = \frac{1}{2} m v_i^2 + m g y_i,
\]
so
\[
v = \sqrt{-2(9.81 \text{ m/s}^2)(-0.084 \text{ m}) - (262 \text{ N/m})(-0.084 \text{ m})^2/(1.25 \text{ kg})} = 0.41 \text{ m/s}.
\]

E13-4 The car climbs a vertical distance of \( (225 \text{ m}) \sin(10^\circ) = 39.1 \text{ m} \) in coming to a stop. The change in energy of the car is then
\[
\Delta E = -\frac{1}{2} \frac{(16400 \text{ N})}{(9.81 \text{ m/s}^2)}[(31.4 \text{ m/s})^2 + (16400 \text{ N})(39.1 \text{ m})] = -1.83 \times 10^5 \text{ J}.
\]

E13-5 (a) Applying conservation of energy to the points where the ball was dropped and where it entered the oil,
\[
\frac{1}{2} m v_i^2 + m g y_i = \frac{1}{2} m v^2 + m g y,
\]
\[
\frac{1}{2} v^2 + g(0) = \frac{1}{2} (0)^2 + g y_i,
\]
\[
v = \sqrt{2 g y_i},
\]
\[
= \sqrt{2(9.81 \text{ m/s}^2)(0.76 \text{ m})} = 3.9 \text{ m/s}.
\]
(b) The change in internal energy of the ball + oil can be found by considering the points where the ball was released and where the ball reached the bottom of the container.
\[
\Delta E = K_f + U_f - K_i - U_i,
\]
\[
= \frac{1}{2} m v^2 + m g y_i - \frac{1}{2} m (0)^2 - m g y,
\]
\[
= \frac{1}{2} (12.2 \times 10^{-3} \text{ kg})(1.48 \text{ m/s})^2 - (12.2 \times 10^{-3} \text{ kg})(9.81 \text{ m/s}^2)(0.55 \text{ m} - 0.76 \text{ m}),
\]
\[
= -0.143 \text{ J}
\]
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E13-6  (a) $U_i = (25.3 \text{ kg})(9.81 \text{ m/s}^2)(12.2 \text{ m}) = 3030 \text{ J}.$
(b) $K_f = \frac{1}{2}(25.3 \text{ kg})(5.56 \text{ m/s})^2 = 391 \text{ J}.$
(c) $\Delta E_{\text{int}} = 3030 \text{ J} - 391 \text{ J} = 2640 \text{ J}.$

E13-7  (a) At atmospheric entry the kinetic energy is

$$K = \frac{1}{2}(7.9 \times 10^4 \text{ kg})(8.0 \times 10^3 \text{ m/s})^2 = 2.5 \times 10^{12} \text{ J}.$$  

The gravitational potential energy is

$$U = (7.9 \times 10^4 \text{ kg})(9.8 \text{ m/s}^2)(1.6 \times 10^5 \text{ m}) = 1.2 \times 10^{11} \text{ J}.$$  

The total energy is $2.6 \times 10^{12} \text{ J}.$

(b) At touch down the kinetic energy is

$$K = \frac{1}{2}(7.9 \times 10^4 \text{ kg})(9.8 \times 10^3 \text{ m/s})^2 = 3.8 \times 10^8 \text{ J}.$$  

E13-8  $\Delta E/\Delta t = (68 \text{ kg})(9.8 \text{ m/s}^2)(59 \text{ m/s}) = 39000 \text{ J/s}.$

E13-9  Let $m$ be the mass of the water under consideration. Then the percentage of the potential energy “lost” which appears as kinetic energy is

$$\frac{K_f - K_i}{U_i - U_f}.$$  

Then

$$\frac{K_f - K_i}{U_i - U_f} = \frac{1}{2} m (v_f^2 - v_i^2) / (mgy_i - mgy_f),$$

$$= \frac{v_f^2 - v_i^2}{-2g\Delta y},$$

$$= \frac{(13 \text{ m/s})^2 - (3.2 \text{ m/s})^2}{-2(9.81 \text{ m/s}^2)(-15 \text{ m})},$$

$$= 54 \%.$$  

The rest of the energy would have been converted to sound and thermal energy.

E13-10  The change in energy is

$$\Delta E = \frac{1}{2}(524 \text{ kg})(62.6 \text{ m/s})^2 - (524 \text{ kg})(9.81 \text{ m/s}^2)(292 \text{ m}) = 4.74 \times 10^5 \text{ J}.$$  

E13-11  $U_f = K_i - (34.6 \text{ J}).$ Then

$$h = \frac{1}{2} \left( \frac{(7.81 \text{ m/s})^2}{2 (9.81 \text{ m/s}^2)} \right) - \frac{(34.6 \text{ J})}{(4.26 \text{ kg})(9.81 \text{ m/s}^2)} = 2.28 \text{ m}.$$  

which means the distance along the incline is $(2.28 \text{ m})/ \sin(33.0^\circ) = 4.19 \text{ m}.$
E13-12  (a) \( K_f = U_i - U_f \), so

\[
v_f = \sqrt{2(9.81 \text{ m/s}^2)[(862 \text{ m}) - (741 \text{ m})]} = 48.7 \text{ m/s}.
\]

That’s a quick 175 km/h; but the speed at the bottom of the valley is 40% of the speed of sound!

(b) \( \Delta E = U_f - U_i \), so

\[
\Delta E = (54.4 \text{ kg})(9.81 \text{ m/s}^2)[(862 \text{ m}) - (741 \text{ m})] = -6.46 \times 10^4 \text{ J};
\]

which means the internal energy of the snow and skis increased by \( 6.46 \times 10^4 \text{ J} \).

E13-13  The final potential energy is 15% less than the initial kinetic plus potential energy of the ball, so

\[
0.85(K_i + U_i) = U_f.
\]

But \( U_i = U_f \), so \( K_i = 0.15U_f/0.85 \), and then

\[
v_i = \sqrt{\frac{0.15}{0.85}2gh} = \sqrt{2(0.176)(9.81 \text{ m/s}^2)(12.4 \text{ m})} = 6.54 \text{ m/s}.
\]

E13-14  Focus on the potential energy. After the \( n \)th bounce the ball will have a potential energy at the top of the bounce of \( U_n = 0.9U_{n-1} \). Since \( U \propto h \), one can write \( h_n = (0.9)^n h_0 \). Solving for \( n \),

\[
n = \log(h_n/h_0)/\log(0.9) = \log(3 \text{ ft/6 ft})/\log(0.9) = 6.58,
\]

which must be rounded up to 7.

E13-15  Let \( m \) be the mass of the ball and \( M \) be the mass of the block.

The kinetic energy of the ball just before colliding with the block is given by \( K_1 = U_0 \), so \( v_1 = \sqrt{2(9.81 \text{ m/s}^2)(0.687 \text{ m})} = 3.67 \text{ m/s} \).

Momentum is conserved, so if \( v_2 \) and \( v_3 \) are velocities of the ball and block after the collision then \( mv_1 = mv_2 + Mv_3 \). Kinetic energy is not conserved, instead

\[
\frac{1}{2}\left(\frac{1}{2}mv_1^2\right) = \frac{1}{2}mv_2^2 + \frac{1}{2}Mv_3^2.
\]

Combine the energy and momentum expressions to eliminate \( v_3 \):

\[
\begin{align*}
mv_1^2 &= 2mv_2^2 + 2M\left(\frac{m}{M}(v_1 - v_2)\right)^2, \\
Mv_1^2 &= 2Mv_2^2 + 2mv_2^2 - 4mv_1v_2 + 2mv_2^2,
\end{align*}
\]

which can be formed into a quadratic. The solution for \( v_2 \) is

\[
v_2 = \frac{2m \pm \sqrt{2(M^2 - mM)}}{2(M + m)}v_1 = (0.600 \pm 1.95) \text{ m/s}.
\]

The corresponding solutions for \( v_3 \) are then found from the momentum expression to be \( v_3 = 0.981 \text{ m/s} \) and \( v_3 = 0.219 \). Since it is unlikely that the ball passed through the block we can toss out the second set of answers.

E13-16  \( E_f = K_f + U_f = 3mgh \), or

\[
v_f = \sqrt{2(9.81 \text{ m/s}^2)(2)(0.18 \text{ m})} = 2.66 \text{ m/s}.
\]
We can find the kinetic energy of the center of mass of the woman when her feet leave the ground by considering energy conservation and her highest point. Then

\[
\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f,
\]

\[
\frac{1}{2}mv_i = mg\Delta y,
\]

\[
= (55.0 \text{ kg})(9.81 \text{ m/s}^2)(1.20 \text{ m} - 0.90 \text{ m}) = 162 \text{ J}.
\]

(a) During the jumping phase her potential energy changed by

\[
\Delta U = mg\Delta y = (55.0 \text{ kg})(9.81 \text{ m/s}^2)(0.50 \text{ m}) = 270 \text{ J}
\]

while she was moving up. Then

\[
F_{\text{ext}} = \frac{\Delta K + \Delta U}{\Delta s} = \frac{(162 \text{ J}) + (270 \text{ J})}{0.5 \text{ m}} = 864 \text{ N}.
\]

(b) Her fastest speed was when her feet left the ground,

\[
v = \frac{2K}{m} = \frac{2(162 \text{ J})}{55.0 \text{ kg}} = 2.42 \text{ m/s}.
\]

(b) The ice skater needs to lose \(\frac{1}{2}(116 \text{ kg})(3.24 \text{ m/s})^2 = 609 \text{ J}\) of internal energy.

(a) The average force exerted on the rail is \(F = (609 \text{ J})/(0.340 \text{ m}) = 1790 \text{ N}\).

12.6 km/h is equal to 3.50 m/s; the initial kinetic energy of the car is

\[
\frac{1}{2}(2340 \text{ kg})(3.50 \text{ m/s})^2 = 1.43 \times 10^4 \text{ J}.
\]

(a) The force exerted on the car is \(F = (1.43 \times 10^4 \text{ J})/(0.64 \text{ m}) = 2.24 \times 10^4 \text{ N}\).

(b) The change increase in internal energy of the car is

\[
\Delta E_{\text{int}} = (2.24 \times 10^4 \text{ N})(0.640 \text{ m} - 0.083 \text{ m}) = 1.25 \times 10^4 \text{ J}.
\]

Note that \(v_n^2 = v'_n^2 - 2\bar{\mathbf{v}}'_n \cdot \mathbf{v}_{\text{cm}} + v_{\text{cm}}^2\). Then

\[
K = \sum_n \frac{1}{2} \left( m_nv'_n^2 - 2m_nv'_n \cdot \bar{\mathbf{v}}_{\text{cm}} + m_nv_{\text{cm}}^2 \right),
\]

\[
= \sum_n \frac{1}{2} m_nv'_n^2 - \left( \sum_n m_nv'_n \right) \cdot \bar{\mathbf{v}}_{\text{cm}} + \left( \sum_n \frac{1}{2} m_n \right) v_{\text{cm}}^2,
\]

\[
= K_{\text{int}} - \left( \sum_n m_nv'_n \right) \cdot \bar{\mathbf{v}}_{\text{cm}} + K_{\text{cm}}.
\]

The middle term vanishes because of the definition of velocities relative to the center of mass.
E13-21  Momentum conservation requires $mv_0 = mv + MV$, where the sign indicates the direction. We are assuming one dimensional collisions. Energy conservation requires

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + \frac{1}{2}MV^2 + E.$$  

Combining,

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + \frac{1}{2}M \left( \frac{m}{M}v_0 - \frac{m}{M}v \right)^2 + E,$$

$$Mv_0^2 = Mv^2 + m(v_0 - v)^2 + 2(M/m)E.$$  

Arrange this as a quadratic in $v$,

$$(M + m)v^2 - (2mv_0)v + (2(M/m)E + mv_0^2 - Mv_0^2) = 0.$$  

This will only have real solutions if the discriminant $(b^2 - 4ac)$ is greater than or equal to zero. Then

$$(2mv_0)^2 \geq 4(M + m) \left( 2(M/m)E + mv_0^2 - Mv_0^2 \right)$$  

is the condition for the minimum $v_0$. Solving the equality condition,

$$4m^2v_0^2 = 4(M + m) \left( 2(M/m)E + (m - M)v_0^2 \right),$$

or $M^2v_0^2 = 2(M + m)(M/m)E$. One last rearrangement, and $v_0 = \sqrt{2(M + m)E/(mM)}$.

P13-1  (a) The initial kinetic energy will equal the potential energy at the highest point plus the amount of energy which is dissipated because of air drag.

$$mgh + fh = \frac{1}{2}mv_0^2,$$

$$h = \frac{v_0^2}{2(g + f/m)} = \frac{v_0^2}{2g(1 + f/w)}.$$  

(b) The final kinetic energy when the stone lands will be equal to the initial kinetic energy minus twice the energy dissipated on the way up, so

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_0^2 - 2fh,$$

$$= \frac{1}{2}mv_0^2 - 2f \frac{v_0^2}{2g(1 + f/w)},$$

$$= \left( \frac{m}{2} - \frac{f}{g(1 + f/w)} \right)v_0^2,$$

$$v^2 = \left( 1 - \frac{2f}{w + f} \right)v_0^2,$$

$$v = \left( \frac{w - f}{w + f} \right)^{1/2}v_0.$$  

P13-2  The object starts with $U = (0.234\,\text{kg})(9.81\,\text{m/s}^2)(1.05\,\text{m}) = 2.41\,\text{J}$. It will move back and forth across the flat portion $(2.41\,\text{J})/(0.688\,\text{J}) = 3.50$ times, which means it will come to a rest at the center of the flat part while attempting one last right to left journey.
P13-3  (a) The work done on the block because of friction is 
\[(0.210)(2.41\text{ kg})(9.81\text{ m/s}^2)(1.83\text{ m}) = 9.09\text{ J}.
\]
The energy dissipated because of friction is 
\[9.09\text{ J}/0.83 = 11.0\text{ J}.
\]
(b) The initial speed of the bullet is 
\[v_0 = \frac{M + m}{m} \cdot v = \frac{(2.41\text{ kg}) + (0.00454\text{ kg})}{(0.00454\text{ kg})}(3.02\text{ m/s}) = 1.60 \times 10^3 \text{ m/s}.
\]

P13-4  The energy stored in the spring after compression is 
\[
\frac{1}{2}(193\text{ N/m})(0.0416\text{ m})^2 = 0.167\text{ J}.
\]
Since 117 mJ was dissipated by friction, the kinetic energy of the block before colliding with the spring was 0.284 J. The speed of the block was then 
\[v = \sqrt{2(0.284\text{ J})/(1.34\text{ kg})} = 0.651 \text{ m/s}.
\]

P13-5  (a) Using Newton’s second law, \(F = ma\), so for circular motion around the proton 
\[\frac{mv^2}{r} = F = \frac{ke^2}{r^2}.
\]
The kinetic energy of the electron in an orbit is then 
\[K = \frac{1}{2}mv^2 = \frac{1}{2}k\frac{e^2}{r}.
\]
The change in kinetic energy is 
\[\Delta K = \frac{1}{2}ke^2 \left(\frac{1}{r_2} - \frac{1}{r_1}\right).
\]
(b) The potential energy difference is 
\[\Delta U = -\int_{r_1}^{r_2} \frac{ke^2}{r^2}dr = -ke^2 \left(\frac{1}{r_2} - \frac{1}{r_1}\right).
\]
(c) The total energy change is 
\[\Delta E = \Delta K + \Delta U = -\frac{1}{2}ke^2 \left(\frac{1}{r_2} - \frac{1}{r_1}\right).
\]

P13-6  (a) The initial energy of the system is \((4000\text{ lb})(12\text{ ft}) = 48,000 \text{ ft} \cdot \text{lb}\). The safety device removes \((1000\text{ lb})(12\text{ ft}) = 12,000 \text{ ft} \cdot \text{lb}\) before the elevator hits the spring, so the elevator has a kinetic energy of \(36,000 \text{ ft} \cdot \text{lb}\) when it hits the spring. The speed of the elevator when it hits the spring is 
\[v = \sqrt{\frac{2(36,000 \text{ ft} \cdot \text{lb})(32.0 \text{ ft/s}^2)}{(4000 \text{ lb})}} = 24.0 \text{ ft/s}.
\]
(b) Assuming the safety clamp remains in effect while the elevator is in contact with the spring then the distance compressed will be found from 
\[36,000 \text{ ft} \cdot \text{lb} = \frac{1}{2}(10,000 \text{ lb/ft})y^2 - (4000 \text{ lb})y + (1000 \text{ lb})y.
\]
This is a quadratic expression in \(y\) which can be simplified to look like

\[5y^2 - 3y - 36 = 0,\]

which has solutions \(y = (0.3 \pm 2.7)\) ft. Only \(y = 3.00\) ft makes sense here.

(c) The distance through which the elevator will bounce back up is found from

\[33,000 \text{ ft} = (4000 \text{ lb})y - (1000 \text{ lb})y,\]

where \(y\) is measured from the most compressed point of the spring. Then \(y = 11\) ft, or the elevator bounces back up 8 feet.

(d) The elevator will bounce until it has traveled a total distance so that the safety device dissipates all of the original energy, or 48 ft.

**P13-7** The net force on the top block while it is being pulled is

\[11.0 \text{ N} - F_f = 11.0 \text{ N} - (0.35)(2.5 \text{ kg})(9.81 \text{ m/s}^2) = 2.42 \text{ N}.\]

This means it is accelerating at \((2.42 \text{ N})/(2.5 \text{ kg}) = 0.968 \text{ m/s}^2\). That acceleration will last a time \(t = \sqrt{2(0.30 \text{ m})/(0.968 \text{ m/s}^2)} = 0.787 \text{ s}\). The speed of the top block after the force stops pulling is then \((0.968 \text{ m/s}^2)(0.787 \text{ s}) = 0.762 \text{ m/s}\). The force on the bottom block is \(F_f\), so the acceleration of the bottom block is

\[(0.35)(2.5 \text{ kg})(9.81 \text{ m/s}^2)/(10.0 \text{ kg}) = 0.858 \text{ m/s}^2,\]

and the speed after the force stops pulling on the top block is \((0.858 \text{ m/s}^2)(0.787 \text{ s}) = 0.675 \text{ m/s}\).

(a) \(W = Fs = (11.0 \text{ N})(0.30 \text{ m}) = 3.3 \text{ J}\) of energy were delivered to the system, but after the force stops pulling only

\[\frac{1}{2}(2.5 \text{ kg})(0.762 \text{ m/s})^2 + \frac{1}{2}(10.0 \text{ kg})(0.675 \text{ m/s})^2 = 3.004 \text{ J}\]

were present as kinetic energy. So 0.296 J is “missing” and would be now present as internal energy.

(b) The impulse received by the two block system is then \(J = (11.0 \text{ N})(0.787 \text{ s}) = 8.66 \text{ N-s}\). This impulse causes a change in momentum, so the speed of the two block system after the external force stops pulling and both blocks move as one is \((8.66 \text{ N-s})(12.5 \text{ kg}) = 0.693 \text{ m/s}\). The final kinetic energy is

\[\frac{1}{2}(12.5 \text{ kg})(0.693 \text{ m/s})^2 = 3.002 \text{ J};\]

this means that 0.002 J are dissipated.

**P13-8** Hmm.