E11-1   (a) Apply Eq. 11-2, $W = Fs \cos \phi = (190\,\text{N})(3.3\,\text{m}) \cos(22°) = 580\,\text{J}$.

(b) The force of gravity is perpendicular to the displacement of the crate, so there is no work done by the force of gravity.

(c) The normal force is perpendicular to the displacement of the crate, so there is no work done by the normal force.

E11-2   (a) The force required is $F = ma = (106\,\text{kg})(1.97\,\text{m/s}^2) = 209\,\text{N}$. The object moves with an average velocity $v_{\text{av}} = v_0/2$ in a time $t = v_0/a$ through a distance $x = v_{\text{av}}t = v_0^2/(2a)$. So

$$x = (51.3\,\text{m/s})^2/[2(1.97\,\text{m/s}^2)] = 668\,\text{m}.$$

The work done is $W = Fx = (-209\,\text{N})(668\,\text{m}) = 1.40 \times 10^5\,\text{J}$.

(b) The force required is

$$F = ma = (106\,\text{kg})(4.82\,\text{m/s}^2) = 511\,\text{N}.$$

$x = (51.3\,\text{m/s})^2/[2(4.82\,\text{m/s}^2)] = 273\,\text{m}$. The work done is $W = Fx = (-511\,\text{N})(273\,\text{m}) = 1.40 \times 10^5\,\text{J}$.

E11-3   (a) $W = Fx = (120\,\text{N})(3.6\,\text{m}) = 430\,\text{J}$.

(b) $W = Fx \cos \theta = mgx \cos \theta = (25\,\text{kg})(9.8\,\text{m/s}^2)(3.6\,\text{m}) \cos(117°) = 400\,\text{N}$.

(c) $W = Fx \cos \theta$, but $\theta = 90°$, so $W = 0$.

E11-4   The worker pushes with a force $\vec{\mathbf{P}}$; this force has components $P_x = P \cos \theta$ and $P_y = P \sin \theta$, where $\theta = -32.0°$. The normal force of the ground on the crate is $N = mg - P_y$, so the force of friction is $f = \mu_k N = \mu_k(mg - P_y)$. The crate moves at constant speed, so $P_x = f$. Then

$$
\begin{aligned}
P \cos \theta &= \mu_k(mg - P \sin \theta), \\
P &= \frac{\mu_k mg}{\cos \theta + \mu_k \sin \theta}.
\end{aligned}
$$

The work done on the crate is

$$
\begin{aligned}
W &= \vec{\mathbf{P}} \cdot \vec{\mathbf{x}} = Px \cos \theta = \frac{\mu_k x mg}{1 + \mu_k \tan \theta}, \\
&= \frac{(0.21)(31.3\,\text{ft})(58.7\,\text{lb})}{1 + (0.21) \tan(-32.0°)} = 444\,\text{ft} \cdot \text{lb}.
\end{aligned}
$$

E11-5   The components of the weight are $W_{||} = mg \sin \theta$ and $W_\perp = mg \cos \theta$. The push $\vec{\mathbf{P}}$ has components $P_{||} = P \cos \theta$ and $P_\perp = P \sin \theta$.

The normal force on the trunk is $N = W_\perp + P\perp$ so the force of friction is $f = \mu_k(mg \cos \theta + P \sin \theta)$. The push required to move the trunk up at constant speed is then found by noting that $P_{||} = W_{||} + f$.

Then

$$P = \frac{mg(\tan \theta + \mu_k)}{1 - \mu_k \tan \theta}.$$

(a) The work done by the applied force is

$$W = Px \cos \theta = \frac{(52.3\,\text{kg})(9.81\,\text{m/s}^2)[\sin(28.0°) + (0.19)\cos(28.0°)](5.95\,\text{m})}{1 - (0.19)\tan(28.0°)} = 2160\,\text{J}.$$

(b) The work done by the force of gravity is

$$W = mgx \cos(\theta + 90°) = (52.3\,\text{kg})(9.81\,\text{m/s}^2)(5.95\,\text{m}) \cos(118°) = -1430\,\text{J}.$$

E11-6 $\theta = \arcsin(0.902\,\text{m}/1.62\,\text{m}) = 33.8°$.

The components of the weight are $W_{||} = mg\sin\theta$ and $W_\perp = mg\cos\theta$.

The normal force on the ice is $N = W_\perp$ so the force of friction is $f = \mu_k mg\cos\theta$. The push required to allow the ice to slide down at constant speed is then found by noting that $P = W_{||} - f$. Then $P = mg(\sin\theta - \mu_k\cos\theta)$.

(a) $P = (47.2\,\text{kg})(9.81\,\text{m/s}^2)[\sin(33.8°) - (0.110)\cos(33.8°)] = 215\,\text{N}$.

(b) The work done by the applied force is $W = Px = (215\,\text{N})(-1.62\,\text{m}) = -348\,\text{J}$.

(c) The work done by the force of gravity is

$$W = mgx\cos(90° - \theta) = (47.2\,\text{kg})(9.81\,\text{m/s}^2)(1.62\,\text{m})\cos(56.2°) = 417\,\text{J}.$$

E11-7 Equation 11-5 describes how to find the dot product of two vectors from components,

$$\begin{aligned}\vec{\mathbf{a}}\cdot\vec{\mathbf{b}} &= a_x b_x + a_y b_y + a_z b_z, \\ &= (3)(2) + (3)(1) + (3)(3) = 18.\end{aligned}$$

Equation 11-3 can be used to find the angle between the vectors,

$$\begin{aligned}a &= \sqrt{(3)^2 + (3)^2 + (3)^2} = 5.19, \\ b &= \sqrt{(2)^2 + (1)^2 + (3)^2} = 3.74.\end{aligned}$$

Now use Eq. 11-3,

$$\cos\phi = \frac{\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}}{ab} = \frac{(18)}{(5.19)(3.74)} = 0.927,$$

and then $\phi = 22.0°$.

E11-8 $\vec{\mathbf{a}}\cdot\vec{\mathbf{b}} = (12)(5.8)\cos(55°) = 40$.

E11-9 $\vec{\mathbf{r}}\cdot\vec{\mathbf{s}} = (4.5)(7.3)\cos(320° - 85°) = -19$.

E11-10 (a) Add the components individually:

$$\vec{\mathbf{r}} = (5 + 2 + 4)\hat{\mathbf{i}} + (4 - 2 + 3)\hat{\mathbf{j}} + (-6 - 3 + 2)\hat{\mathbf{k}} = 11\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 7\hat{\mathbf{k}}.$$

(b) $\theta = \arccos(-7/\sqrt{11^2 + 5^2 + 7^2}) = 120°$.

(c) $\theta = \arccos(\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}/\sqrt{ab})$, or

$$\theta = \frac{(5)(-2) + (4)(2) + (-6)(3)}{\sqrt{(5^2 + 4^2 + 6^2)(2^2 + 2^2 + 3^2)}} = 124°.$$

E11-11 There are two forces on the woman, the force of gravity directed down and the normal force of the floor directed up. These will be effectively equal, so $N = W = mg$. Consequently, the 57 kg woman must exert a force of $F = (57\text{kg})(9.8\text{m/s}^2) = 560\text{N}$ to propel herself up the stairs.

From the reference frame of the woman the stairs are moving down, and she is exerting a force down, so the work done by the woman is given by

$$W = Fs = (560\text{ N})(4.5\text{ m}) = 2500\text{ J},$$

this work is positive because the force is in the same direction as the displacement.

The average power supplied by the woman is given by Eq. 11-7,

$$P = W/t = (2500\text{ J})/(3.5\text{ s}) = 710\text{ W}.$$

E11-12 $P = W/t = mgy/t = (100 \times 667\,\text{N})(152\,\text{m})/(55.0\,\text{s}) = 1.84\times10^5\text{W}$.

E11-13 $P = Fv = (110\,\text{N})(0.22\,\text{m/s}) = 24\,\text{W}$.

E11-14 $F = P/v$, but the units are awkward.

$$F = \frac{(4800\text{ hp})}{(77\text{ knots})}\frac{1\text{ knot}}{1.688\text{ ft/s}}\frac{1\text{ ft/s}}{0.3048\text{ m/s}}\frac{745.7\text{ W}}{1\text{ hp}} = 9.0\times10^4\text{N}.$$

E11-15 $P = Fv = (720\text{ N})(26\text{ m/s}) = 19000\text{ W}$; in horsepower, $P = 19000\text{ W}(1/745.7\text{ hp/W}) = 25$ hp.

E11-16 Change to metric units! Then $P = 4920\,\text{W}$, and the flow rate is $Q = 13.9\text{ L/s}$. The density of water is approximately 1.00 kg/L , so the mass flow rate is $R = 13.9$ kg/s.

$$y = \frac{P}{gR} = \frac{(4920\,\text{kg})}{(9.81\,\text{m/s}^2)(13.9\text{ kg/s})} = 36.1\,\text{m},$$

which is the same as approximately 120 feet.

E11-17 (a) Start by converting kilowatt-hours to Joules:

$$1\text{ kW} \cdot \text{h} = (1000\text{ W})(3600\text{ s}) = 3.6\times10^6\text{ J}.$$

The car gets 30 mi/gal, and one gallon of gas produces 140 MJ of energy. The gas required to produce 3.6×10^6J is

$$3.6\times10^6\text{J}\left(\frac{1\text{ gal}}{140\times10^6\text{J}}\right) = 0.026\text{ gal}.$$

The distance traveled on this much gasoline is

$$0.026\text{ gal}\left(\frac{30\text{ mi}}{1\text{ gal}}\right) = 0.78\text{ mi}.$$

(b) At 55 mi/h, it will take

$$0.78\text{ mi}\left(\frac{1\text{ hr}}{55\text{ mi}}\right) = 0.014\text{ h} = 51\text{ s}.$$

The rate of energy expenditure is then $(3.6\times10^6\text{ J})/(51\text{ s}) = 71000\text{ W}$.

E11-18 The linear speed is $v = 2\pi(0.207\,\text{m})(2.53\text{ rev/s}) = 3.29\,\text{m/s}$. The frictional force is $f = \mu_k N = (0.32)(180\,\text{N}) = 57.6\,\text{N}$. The power developed is $P = Fv = (57.6\,\text{N})(3.29\,\text{m}) = 190\,\text{W}$.

E11-19 The net force required will be $(1380\,\text{kg} - 1220\,\text{kg})(9.81\,\text{m/s}^2) = 1570\,\text{N}$. The work is $W = Fy$, the power output is $P = W/t = (1570\,\text{N})(54.5\,\text{m})/(43.0\,\text{s}) = 1990\,\text{W}$, or $P = 2.67$ hp.

E11-20 (a) The momentum change of the ejected material in one second is

$$\Delta p = (70.2\,\text{kg})(497\,\text{m/s} - 184\,\text{m/s}) + (2.92\,\text{kg})(497\,\text{m/s}) = 2.34\times10^4\text{kg} \cdot \text{m/s}.$$

The thrust is then $F = \Delta p/\Delta t = 2.34\times10^4\text{N}$.
  (b) The power is $P = Fv = (2.34\times10^4\text{ N})(184\,\text{m/s}) = 4.31\times10^6\text{W}$. That's 5780 hp.

E11-21 The acceleration on the object as a function of position is given by

$$a = \frac{20 \text{ m/s}^2}{8 \text{ m}} x,$$

The work done on the object is given by Eq. 11-14,

$$W = \int_0^8 F_x dx = \int_0^8 (10 \text{ kg}) \frac{20 \text{ m/s}^2}{8 \text{ m}} x \, dx = 800 \text{ J}.$$

E11-22 Work is area between the curve and the line $F = 0$. Then

$$W = (10 \text{ N})(2 \text{ s}) + \frac{1}{2}(10 \text{ N})(2 \text{ s}) + \frac{1}{2}(-5 \text{ N})(2 \text{ s}) = 25 \text{ J}.$$

E11-23 (a) For a spring, $F = -kx$, and $\Delta F = -k\Delta x$.

$$k = -\frac{\Delta F}{\Delta x} = -\frac{(-240 \text{ N}) - (-110 \text{ N})}{(0.060 \text{ m}) - (0.040 \text{ m})} = 6500 \text{ N/m}.$$

With no force on the spring,

$$\Delta x = -\frac{\Delta F}{k} = -\frac{(0) - (-110 \text{ N})}{(6500 \text{ N/m})} = -0.017 \text{ m}.$$

This is the amount *less* than the 40 mm mark, so the position of the spring with no force on it is 23 mm.

(b) $\Delta x = -10$ mm compared to the 100 N picture, so

$$\Delta F = -k\Delta x = -(6500 \text{ N/m})(-0.010 \text{ m}) = 65 \text{ N}.$$

The weight of the last object is $110 \text{ N} - 65 \text{ N} = 45 \text{ N}$.

E11-24 (a) $W = \frac{1}{2}k(x_\text{f}^2 - x_\text{i}^2) = \frac{1}{2}(1500 \text{ N/m})(7.60 \times 10^{-3} \text{m})^2 = 4.33 \times 10^{-2} \text{J}$.
(b) $W = \frac{1}{2}(1500 \text{ N/m})[(1.52 \times 10^{-2} \text{m})^2 - (7.60 \times 10^{-3} \text{m})^2 = 1.30 \times 10^{-1} \text{J}$.

E11-25 Start with Eq. 11-20, and let $F_x = 0$ while $F_y = -mg$:

$$W = \int_i^f (F_x dx + F_y dy) = -mg \int_i^f dy = -mgh.$$

E11-26 (a) $F_0 = mv_0^2/r_0 = (0.675 \text{ kg})(10.0 \text{ m/s})^2/(0.500 \text{ m}) = 135 \text{ N}$.
(b) Angular momentum is conserved, so $v = v_0(r_0/r)$. The force then varies as $F = mv^2/r = mv_0^2 r_0^2/r^3 = F_0(r_0/r)^3$. The work done is

$$W = \int \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = \frac{(-135 \text{ N})(0.500 \text{ m})^3}{-2} \left((0.300 \text{ m})^{-2} - (0.500 \text{ m})^{-2} \right) = 60.0 \text{ J}.$$

E11-27 The kinetic energy of the electron is

$$4.2 \text{ eV} \left(\frac{1.60 \times 10^{-19} \text{ J}}{1 \text{ eV}} \right) = 6.7 \times 10^{-19} \text{ J}.$$

Then

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(6.7 \times 10^{-19} \text{J})}{(9.1 \times 10^{-31} \text{kg})}} = 1.2 \times 10^6 \text{m/s}.$$

E11-28 (a) $K = \frac{1}{2}(110\,\text{kg})(8.1\,\text{m/s})^2 = 3600\,\text{J}$.
  (b) $K = \frac{1}{2}(4.2\times10^{-3}\text{kg})(950\,\text{m/s})^2 = 1900\,\text{J}$.
  (c) $m = 91,400\text{ tons}(907.2\,\text{kg/ton}) = 8.29\times10^7\text{kg}$;

$$v = 32.0\text{ knots}(1.688\text{ ft/s/knot})(0.3048\text{ m/ft}) = 16.5\,\text{m/s}.$$

$$K = \frac{1}{2}(8.29\times10^7\text{kg})(16.5\,\text{m/s})^2 = 1.13\times10^{10}\text{J}.$$

E11-29 (b) $\Delta K = W = Fx = (1.67\times10^{-27}\text{kg})(3.60\times10^{15}\text{m/s}^2)(0.0350\,\text{m}) = 2.10\times10^{-13}\text{J}$. That's
$2.10\times10^{-13}\text{J}/(1.60\times10^{-19}\text{J/eV}) = 1.31\times10^6\text{eV}$.
  (a) $K_\text{f} = 2.10\times10^{-13}\text{J} + \frac{1}{2}(1.67\times10^{-27}\text{kg})(2.40\times10^7\text{m/s}^2) = 6.91\times10^{-13}\text{J}$. Then

$$v_\text{f} = \sqrt{2K/m} = \sqrt{2(6.91\times10^{-13}\text{J})/(1.67\times10^{-27}\text{kg})} = 2.88\times10^7\text{m/s}.$$

E11-30 Work is negative if kinetic energy is decreasing. This happens only in region CD. The work is zero in region BC. Otherwise it is positive.

E11-31 (a) Find the velocity of the particle by taking the time derivative of the position:

$$v = \frac{dx}{dt} = (3.0\,\text{m/s}) - (8.0\,\text{m/s}^2)t + (3.0\,\text{m/s}^3)t^2.$$

Find v at two times: $t = 0$ and $t = 4\,\text{s}$.

$$\begin{aligned}
v(0) &= (3.0\,\text{m/s}) - (8.0\,\text{m/s}^2)(0) + (3.0\,\text{m/s}^3)(0)^2 = 3.0\,\text{m/s}, \\
v(4) &= (3.0\,\text{m/s}) - (8.0\,\text{m/s}^2)(4.0\,\text{s}) + (3.0\,\text{m/s}^3)(4.0\,\text{s})^2 = 19.0\,\text{m/s}
\end{aligned}$$

The initial kinetic energy is $K_\text{i} = \frac{1}{2}(2.80\,\text{kg})(3.0\,\text{m/s})^2 = 13\,\text{J}$, while the final kinetic energy is $K_\text{f} = \frac{1}{2}(2.80\,\text{kg})(19.0\,\text{m/s})^2 = 505\,\text{J}$.
  The work done by the force is given by Eq. 11-24,

$$W = K_\text{f} - K_\text{i} = 505\,\text{J} - 13\,\text{J} = 492\,\text{J}.$$

(b) This question is asking for the instantaneous power when $t = 3.0\,\text{s}$. $P = Fv$, so first find a;

$$a = \frac{dv}{dt} = -(8.0\,\text{m/s}^2) + (6.0\,\text{m/s}^3)t.$$

Then the power is given by $P = mav$, and when $t = 3\,\text{s}$ this gives

$$P = mav = (2.80\,\text{kg})(10\,\text{m/s}^2)(6\,\text{m/s}) = 168\,\text{W}.$$

E11-32 $W = \Delta K = -K_\text{i}$. Then

$$W = -\frac{1}{2}(5.98\times10^{24}\text{kg})(29.8\times10^3\text{m/s})^2 = 2.66\times10^{33}\text{J}.$$

E11-33 (a) $K = \frac{1}{2}(1600\,\text{kg})(20\,\text{m/s})^2 = 3.2\times10^5\text{J}$.
  (b) $P = W/t = (3.2\times10^5\text{J})/(33\,\text{s}) = 9.7\times10^3\text{W}$.
  (c) $P = Fv = mav = (1600\,\text{kg})(20\,\text{m/s}/33.0\,\text{s})(20\,\text{m/s}) = 1.9\times10^4\text{W}$.

E11-34 (a) $I = 1.40\times10^4\text{u} \cdot \text{pm}^2(1.66\times10^{-27}\text{kg}/mboxu) = 2.32\times10^{-47}\text{kg} \cdot \text{m}^2$.
  (b) $K = \frac{1}{2}I\omega^2 = \frac{1}{2}(2.32\times10^{-47}\text{kg} \cdot \text{m}^2)(4.30\times10^{12}\text{ rad/s})^2 = 2.14\times10^{-22}\text{J}$. That's 1.34 meV.

E11-35 The translational kinetic energy is $K_t = \frac{1}{2}mv^2$, the rotational kinetic energy is $K_r = \frac{1}{2}I\omega^2 = \frac{2}{3}K_t$. Then

$$\omega = \sqrt{\frac{2m}{3I}}v = \sqrt{\frac{2(5.30\times 10^{-26}\text{kg})}{3(1.94\times 10^{-46}\text{kg}\cdot\text{m}^2)}}(500\,\text{m/s}) = 6.75\times 10^{12}\text{ rad/s}.$$

E11-36 $K_r = \frac{1}{2}I\omega^2 = \frac{1}{4}(512\,\text{kg})(0.976\,\text{m})^2(624\text{ rad/s})^2 = 4.75\times 10^7\text{J}.$
(b) $t = W/P = (4.75\times 10^7\text{J})/(8130\,\text{W}) = 5840\,\text{s}$, or 97.4 minutes.

E11-37 From Eq. 11-29, $K_i = \frac{1}{2}I\omega_i^2$. The object is a hoop, so $I = MR^2$. Then

$$K_i = \frac{1}{2}MR^2\omega^2 = \frac{1}{2}(31.4\,\text{kg})(1.21\,\text{m})^2(29.6\,\text{rad/s})^2 = 2.01\times 10^4\text{ J}.$$

Finally, the average power required to stop the wheel is

$$P = \frac{W}{t} = \frac{K_f - K_i}{t} = \frac{(0) - (2.01\times 10^4\text{ J})}{(14.8\,\text{s})} = -1360\,\text{W}.$$

E11-38 The wheels are connected by a belt, so $r_A\omega_A = r_B\omega_B$, or $\omega_A = 3\omega_B$.
(a) If $l_A = l_B$ then

$$\frac{I_A}{I_B} = \frac{l_A/\omega_A}{l_B/\omega_B} = \frac{\omega_B}{\omega_A} = \frac{1}{3}.$$

(b) If instead $K_A = K_B$ then

$$\frac{I_A}{I_B} = \frac{2K_A/\omega_A{}^2}{2K_B/\omega_B{}^2} = \frac{\omega_B{}^2}{\omega_A{}^2} = \frac{1}{9}.$$

E11-39 (a) $\omega = 2\pi/T$, so

$$K = \frac{1}{2}I\omega^2 = \frac{4\pi^2}{5}\frac{(5.98\times 10^{24}\text{kg})(6.37\times 10^6\text{m})^2}{(86,400\,\text{s})^2} = 2.57\times 10^{29}\text{J}$$

(b) $t = (2.57\times 10^{29}\text{J})/(6.17\times 10^{12}\text{W}) = 4.17\times 10^{16}\text{s}$, or 1.3 billion years.

E11-40 (a) The velocities relative to the center of mass are $m_1v_1 = m_2v_2$; combine with $v_1 + v_2 = 910.0\,\text{m/s}$ and get

$$(290.0\,\text{kg})v_1 = (150.0\,\text{kg})(910\,\text{m/s} - v_1),$$

or

$$v_1 = (150\,\text{kg})(910\,\text{m/s})/(290\,\text{kg} + 150\,\text{kg}) = 310\,\text{m/s}$$

and $v_2 = 600\,\text{m/s}$. The rocket case was sent back, so $v_c = 7600\,\text{m/s} - 310\,\text{m/s} = 7290\,\text{m/s}$. The payload capsule was sent forward, so $v_p = 7600\,\text{m/s} + 600\,\text{m/s} = 8200\,\text{m/s}$.
(b) Before,

$$K_i = \frac{1}{2}(290\,\text{kg} + 150\,\text{kg})(7600\,\text{m/s})^2 = 1.271\times 10^{10}\text{J}.$$

After,

$$K_f = \frac{1}{2}(290\,\text{kg})(7290\,\text{m/s})^2 + \frac{1}{2}(150\,\text{kg})(8200\,\text{m/s})^2 = 1.275\times 10^{10}\text{J}.$$

The "extra" energy came from the spring.

E11-41 Let the mass of the freight car be M and the initial speed be v_i. Let the mass of the caboose be m and the final speed of the coupled cars be v_f. The caboose is originally at rest, so the expression of momentum conservation is

$$Mv_i = Mv_f + mv_f = (M + m)v_f$$

The decrease in kinetic energy is given by

$$\begin{aligned} K_i - K_f &= \frac{1}{2}Mv_i{}^2 - \left(\frac{1}{2}Mv_f{}^2 + \frac{1}{2}mv_f{}^2\right), \\ &= \frac{1}{2}\left(Mv_i{}^2 - (M+m)v_f{}^2\right) \end{aligned}$$

What we really want is $(K_i - K_f)/K_i$, so

$$\begin{aligned} \frac{K_i - K_f}{K_i} &= \frac{Mv_i{}^2 - (M+m)v_f{}^2}{Mv_i{}^2}, \\ &= 1 - \frac{M+m}{M}\left(\frac{v_f}{v_i}\right)^2, \\ &= 1 - \frac{M+m}{M}\left(\frac{M}{M+m}\right)^2, \end{aligned}$$

where in the last line we substituted from the momentum conservation expression.

Then

$$\frac{K_i - K_f}{K_i} = 1 - \frac{M}{M+m} = 1 - \frac{Mg}{Mg + mg}.$$

The left hand side is 27%. We want to solve this for mg, the weight of the caboose. Upon rearranging,

$$mg = \frac{Mg}{1 - 0.27} - Mg = \frac{(35.0\text{ ton})}{(0.73)} - (35.0\text{ ton}) = 12.9\text{ ton}.$$

E11-42 Since the body splits into two parts with equal mass then the velocity gained by one is identical to the velocity "lost" by the other. The initial kinetic energy is

$$K_i = \frac{1}{2}(8.0\text{ kg})(2.0\text{ m/s})^2 = 16\text{ J}.$$

The final kinetic energy is 16 J greater than this, so

$$\begin{aligned} K_f &= 32\text{ J} = \frac{1}{2}(4.0\text{ kg})(2.0\text{ m/s} + v)^2 + \frac{1}{2}(4.0\text{ kg})(2.0\text{ m/s} - v)^2, \\ &= \frac{1}{2}(8.0\text{ kg})[(2.0\text{ m/s})^2 + v^2], \end{aligned}$$

so $16.0\text{ J} = (4.0\text{ kg})v^2$. Then $v = 2.0\text{ m/s}$; one chunk comes to a rest while the other moves off at a speed of 4.0 m/s.

E11-43 The initial velocity of the neutron is $v_0\hat{\mathbf{i}}$, the final velocity is $v_1\hat{\mathbf{j}}$. By momentum conservation the final momentum of the deuteron is $m_n(v_0\hat{\mathbf{i}} - v_1\hat{\mathbf{j}})$. Then $m_d v_2 = m_n\sqrt{v_0^2 + v_1^2}$.

There is also conservation of kinetic energy:

$$\frac{1}{2}m_n v_0^2 = \frac{1}{2}m_n v_1^2 + \frac{1}{2}m_d v_2^2.$$

Rounding the numbers slightly we have $m_d = 2m_n$, then $4v_2^2 = v_0^2 + v_1^2$ is the momentum expression and $v_0^2 = v_1^2 + 2v_2^2$ is the energy expression. Combining,

$$2v_0^2 = 2v_1^2 + (v_0^2 + v_1^2),$$

or $v_1^2 = v_0^2/3$. So the neutron is left with 1/3 of its original kinetic energy.

E11-44 (a) The third particle must have a momentum

$$\vec{p}_3 = -(16.7\times10^{-27}\text{kg})(6.22\times10^6\text{m/s})\hat{i} + (8.35\times10^{-27}\text{kg})(7.85\times10^6\text{m/s})\hat{j}$$
$$= (-1.04\hat{i} + 0.655\hat{j})\times10^{-19}\text{kg}\cdot\text{m/s}.$$

(b) The kinetic energy can also be written as $K = \frac{1}{2}mv^2 = \frac{1}{2}m(p/m)^2 = p^2/2m$. Then the kinetic energy appearing in this process is

$$K = \frac{1}{2}(16.7\times10^{-27}\text{kg})(6.22\times10^6\text{m/s})^2 + \frac{1}{2}(8.35\times10^{-27}\text{kg})(7.85\times10^6\text{m/s})^2$$
$$+ \frac{1}{2(11.7\times10^{-27}\text{kg})}(1.23\times10^{-19}\text{kg}\cdot\text{m/s})^2 = 1.23\times10^{-12}\text{J}.$$

This is the same as 7.66 MeV.

P11-1 Change your units! Then

$$F = \frac{W}{s} = \frac{(4.5\text{ eV})(1.6\times10^{-19}\text{ J/eV})}{(3.4\times10^{-9}\text{ m})} = 2.1\times10^{-10}\text{ N}.$$

P11-2 (a) If the acceleration is $-g/4$ the the net force on the block is $-Mg/4$, so the tension in the cord must be $T = 3Mg/4$.
(a) The work done by the cord is $W = \vec{F}\cdot\vec{s} = (3Mg/4)(-d) = -(3/4)Mgd$.
(b) The work done by gravity is $W = \vec{F}\cdot\vec{s} = (-Mg)(-d) = Mgd$.

P11-3 (a) There are *four* cords which are attached to the bottom load L. Each supports a tension F, so to lift the load $4F = (840\text{ lb}) + (20.0\text{ lb})$, or $F = 215$ lb.
(b) The work done against gravity is $W = \vec{F}\cdot\vec{s} = (840\text{ lb})(12.0\text{ ft}) = 10100$ ft · lb.
(c) To lift the load 12 feet each segment of the cord must be shortened by 12 ft; there are four segments, so the end of the cord must be pulled through a distance of 48.0 ft.
(d) The work done by the applied force is $W = \vec{F}\cdot\vec{s} = (215\text{ lb})(48.0\text{ ft}) = 10300$ ft · lb.

P11-4 The incline has a height h where $h = W/mg = (680\text{ J})/[(75\text{ kg})(9.81\text{ m/s}^2)]$. The work required to lift the block is the same regardless of the path, so the length of the incline l is $l = W/F = (680\text{ J})/(320\text{ N})$. The angle of the incline is then

$$\theta = \arcsin\frac{h}{l} = \arcsin\frac{F}{mg} = \arcsin\frac{(320\text{ N})}{(75\text{ kg})(9.81\text{ m/s}^2)} = 25.8°.$$

P11-5 (a) In 12 minutes the horse moves $x = (5280\text{ ft/mi})(6.20\text{ mi/h})(0.200\text{ h}) = 6550$ ft. The work done by the horse in that time is $W = \vec{F}\cdot\vec{s} = (42.0\text{ lb})(6550\text{ ft})\cos(27.0°) = 2.45\times10^5$ ft · lb.
(b) The power output of the horse is

$$P = \frac{(2.45\times10^5\text{ ft}\cdot\text{lb})}{(720\text{ s})} = 340\text{ ft}\cdot\text{lb/s}\frac{1\text{ hp}}{550\text{ ft}\cdot\text{lb/s}} = 0.618\text{ hp}.$$

P11-6 In this problem $\theta = \arctan(28.2/39.4) = 35.5°$.
The weight of the block has components $W_\| = mg\sin\theta$ and $W_\perp = mg\cos\theta$. The force of friction on the block is $f = \mu_k N = \mu_k mg\cos\theta$. The tension in the rope must then be

$$T = mg(\sin\theta + \mu_k\cos\theta)$$

in order to move the block up the ramp. The power supplied by the winch will be $P = Tv$, so

$$P = (1380\text{ kg})(9.81\text{ m/s}^2)[\sin(35.5°) + (0.41)\cos(35.5°)](1.34\text{ m/s}) = 1.66\times10^4\text{W}.$$

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P11-7 If the power is constant then the force on the car is given by $F = P/v$. But the force is related to the acceleration by $F = ma$ and to the speed by $F = m\frac{dv}{dt}$ for motion in one dimension. Then

$$
\begin{aligned}
F &= \frac{P}{v}, \\
m\frac{dv}{dt} &= \frac{P}{v}, \\
m\frac{dx}{dt}\frac{dv}{dx} &= \frac{P}{v}, \\
mv\frac{dv}{dx} &= \frac{P}{v}, \\
\int_0^v mv^2 dv &= \int_0^x P dx, \\
\frac{1}{3}mv^3 &= Px.
\end{aligned}
$$

We can rearrange this final expression to get v as a function of x, $v = (3xP/m)^{1/3}$.

P11-8 (a) If the drag is $D = bv^2$, then the force required to move the plane forward at constant speed is $F = D = bv^2$, so the power required is $P = Fv = bv^3$.

(b) $P \propto v^3$, so if the speed increases to 125% then P increases by a factor of $1.25^3 = 1.953$, or increases by 95.3%.

P11-9 (a) $P = mgh/t$, but m/t is the persons per minute times the average mass, so

$$P = (100 \text{ people/min})(75.0\text{kg})(9.81 \text{ m/s}^2)(8.20 \text{ m}) = 1.01 \times 10^4 \text{W}.$$

(b) In 9.50 s the Escalator has moved $(0.620 \text{ m/s})(9.50 \text{ s}) = 5.89$ m; so the Escalator has "lifted" the man through a distance of $(5.89 \text{ m})(8.20 \text{ m}/13.3 \text{ m}) = 3.63$ m. The man did the rest himself.

The work done by the Escalator is then $W = (83.5 \text{ kg})(9.81 \text{ m/s}^2)(3.63 \text{ m}) = 2970 \text{ J}$.

(c) Yes, because the point of contact is moving in a direction with at least some non-zero component of the force. The power is

$$P = (83.5 \text{ m/s}^2)(9.81 \text{ m/s}^2)(0.620 \text{ m/s})(8.20 \text{ m}/13.3 \text{ m}) = 313 \text{ W}.$$

(d) If there is a force of contact between the man and the Escalator then the Escalator is doing work on the man.

P11-10 (a) $dP/dv = ab - 3av^2$, so P_{max} occurs when $3v^2 = b$, or $v = \sqrt{b/3}$.

(b) $F = P/v$, so $dF/dv = -2v$, which means F is a maximum when $v = 0$.

(c) No; $P = 0$, but $F = ab$.

P11-11 (b) Integrate,

$$W = \int_0^{3x_0} \vec{\mathbf{F}} \cdot d\vec{\mathbf{s}} = \frac{F_0}{x_0} \int_0^{3x_0} (x - x_0)dx = F_0 x_0 \left(\frac{9}{2} - 3\right),$$

or $W = 3F_0 x_0/2$.

P11-12 (a) Simpson's rule gives

$$W = \frac{1}{3}\left[(10\,\text{N}) + 4(2.4\,\text{N}) + (0.8\,\text{N})\right](1.0\,\text{m}) = 6.8\,\text{J}.$$

(b) $W = \int F\,ds = \int (A/x^2)dx = -A/x$, evaluating this between the limits of integration gives $W = (9\,\text{N}\cdot\text{m}^2)(1/1\,\text{m} - 1/3\,\text{m}) = 6\,\text{J}$.

P11-13 The work required to stretch the spring from x_i to x_f is given by

$$W = \int_{x_i}^{x_f} kx^3 dx = \frac{k}{4}x_f{}^4 - \frac{k}{4}x_i{}^4.$$

The problem gives

$$W_0 = \frac{k}{4}(l)^4 - \frac{k}{4}(0)^4 = \frac{k}{4}l^4.$$

We then want to find the work required to stretch from $x = l$ to $x = 2l$, so

$$
\begin{aligned}
W_{l\to 2l} &= \frac{k}{4}(2l)^4 - \frac{k}{4}(l)^4, \\
&= 16\frac{k}{4}l^4 - \frac{k}{4}l^4, \\
&= 15\frac{k}{4}l^4 = 15W_0.
\end{aligned}
$$

P11-14 (a) The spring extension is $\delta l = \sqrt{l_0^2 + x^2} - l_0$. The force from one spring has magnitude $k\delta l$, but only the x component contributes to the problem, so

$$F = 2k\left(\sqrt{l_0^2 + x^2} - l_0\right)\frac{x}{\sqrt{l_0^2 + x^2}}$$

is the force required to move the point.

The work required is the integral, $W = \int_0^x F\,dx$, which is

$$W = kx^2 - 2kl_0\sqrt{l_0^2 + x^2} + 2kl_0^2$$

Note that it does reduce to the expected behavior for $x \gg l_0$.

(b) Binomial expansion of square root gives

$$\sqrt{l_0^2 + x^2} = l_0\left(1 + \frac{1}{2}\frac{x^2}{l_0^2} - \frac{1}{8}\frac{x^4}{l_0^4}\cdots\right),$$

so the first term in the above expansion cancels with the last term in W; the second term cancels with the first term in W, leaving

$$W = \frac{1}{4}k\frac{x^4}{l_0^2}.$$

P11-15 Number the springs clockwise from the top of the picture. Then the four forces *on* each spring are

$$
\begin{aligned}
F_1 &= k(l_0 - \sqrt{x^2 + (l_0 - y)^2}), \\
F_2 &= k(l_0 - \sqrt{(l_0 - x)^2 + y^2}), \\
F_3 &= k(l_0 - \sqrt{x^2 + (l_0 + y)^2}), \\
F_4 &= k(l_0 - \sqrt{(l_0 + x)^2 + y^2}).
\end{aligned}
$$

The directions are *much* harder to work out, but for small x and y we can assume that

$$\begin{aligned}
\vec{\mathbf{F_1}} &= k(l_0 - \sqrt{x^2 + (l_0 - y)^2})\hat{\mathbf{j}}, \\
\vec{\mathbf{F_2}} &= k(l_0 - \sqrt{(l_0 - x)^2 + y^2})\hat{\mathbf{i}}, \\
\vec{\mathbf{F_3}} &= k(l_0 - \sqrt{x^2 + (l_0 + y)^2})\hat{\mathbf{j}}, \\
\vec{\mathbf{F_4}} &= k(l_0 - \sqrt{(l_0 + x)^2 + y^2})\hat{\mathbf{i}}.
\end{aligned}$$

Then

$$W = \int \vec{\mathbf{F}} \cdot d\vec{\mathbf{s}} = \int (F_1 + F_3)dy + \int (F_2 + F_4)dx,$$

Since x and y are small, expand the force(s) in a binomial expansion:

$$F_1(x, y) \approx F_1(0, 0) + \left.\frac{\partial F_1}{\partial x}\right|_{x,y=0} x + \left.\frac{\partial F_1}{\partial y}\right|_{x,y=0} y = ky;$$

there will be similar expression for the other four forces. Then

$$W = \int 2ky\, dy + \int 2kx\, dx = k(x^2 + y^2) = kd^2.$$

P11-16 (a) $K_i = \frac{1}{2}(1100\,\text{kg})(12.8\,\text{m/s})^2 = 9.0 \times 10^4\,\text{J}$. Removing 51 kJ leaves 39 kJ behind, so

$$v_f = \sqrt{2K_f/m} = \sqrt{2(3.9 \times 10^4\,\text{J})/(1100\,\text{kg})} = 8.4\,\text{m/s},$$

or 30 km/h.
  (b) 39 kJ, as was found above.

 P11-17   Let M be the mass of the man and m be the mass of the boy. Let v_M be the original speed of the man and v_m be the original speed of the boy. Then

$$\frac{1}{2}Mv_M^2 = \frac{1}{2}\left(\frac{1}{2}mv_m^2\right)$$

and

$$\frac{1}{2}M(v_M + 1.0\,\text{m/s})^2 = \frac{1}{2}mv_m^2.$$

  Combine these two expressions and solve for v_M,

$$\begin{aligned}
\frac{1}{2}Mv_M^2 &= \frac{1}{2}\left(\frac{1}{2}M(v_M + 1.0\,\text{m/s})^2\right), \\
v_M^2 &= \frac{1}{2}(v_M + 1.0\,\text{m/s})^2, \\
0 &= -v_M^2 + (2.0\,\text{m/s})v_M + (1.0\,\text{m/s})^2.
\end{aligned}$$

The last line can be solved as a quadratic, and $v_M = (1.0\,\text{m/s}) \pm (1.41\,\text{m/s})$. Now we use the very first equation to find the speed of the boy,

$$\begin{aligned}
\frac{1}{2}Mv_M^2 &= \frac{1}{2}\left(\frac{1}{2}mv_m^2\right), \\
v_M^2 &= \frac{1}{4}v_m^2, \\
2v_M &= v_m.
\end{aligned}$$

P11-18 (a) The work done by gravity on the projectile as it is raised up to 140 m is $W = -mgy = -(0.550\,\text{kg})(9.81\,\text{m/s}^2)(140\,\text{m}) = -755\,\text{J}$. Then the kinetic energy at the highest point is $1550\,\text{J} - 755\,\text{J} = 795\,\text{J}$. Since the projectile must be moving horizontally at the highest point, the horizontal velocity is $v_x = \sqrt{2(795\,\text{J})/(0.550\,\text{kg})} = 53.8\,\text{m/s}$.

(b) The magnitude of the velocity at launch is $v = \sqrt{2(1550\,\text{J})/(0.550\,\text{kg})} = 75.1\,\text{m/s}$. Then $v_y = \sqrt{(75.1\,\text{m/s})^2 - (53.8\,\text{m/s})^2} = 52.4\,\text{m/s}$.

(c) The kinetic energy at that point is $\frac{1}{2}(0.550\,\text{kg})[(65.0\,\text{m/s})^2 + (53.8\,\text{m/s})^2] = 1960\,\text{J}$. Since it has extra kinetic energy, it must be below the launch point, and gravity has done 410 J of work on it. That means it is $y = (410\,\text{J})/[(0.550\,\text{kg})(9.81\,\text{m/s}^2)] = 76.0\,\text{m}$ below the launch point.

P11-19 (a) $K = \frac{1}{2}mv^2 = \frac{1}{2}(8.38 \times 10^{11}\text{kg})(3.0 \times 10^4\text{m/s})^2 = 3.77 \times 10^{20}\text{J}$. In terms of TNT, $K = 9.0 \times 10^4$ megatons.

(b) The diameter will be $\sqrt[3]{8.98 \times 10^4} = 45$ km.

P11-20 (a) $W_\text{g} = -(0.263\,\text{kg})(9.81\,\text{m/s}^2)(-0.118\,\text{m}) = 0.304\,\text{J}$.

(b) $W_\text{s} = -\frac{1}{2}(252\,\text{N/m})(-0.118\,\text{m})^2 = -1.75\,\text{J}$.

(c) The kinetic energy just before hitting the block would be $1.75\,\text{J} - 0.304\,\text{J} = 1.45\,\text{J}$. The speed is then $v = \sqrt{2(1.45\,\text{J})/(0.263\,\text{kg})} = 3.32\,\text{m/s}$.

(d) Doubling the speed quadruples the initial kinetic energy to 5.78 J. The compression will then be given by

$$-5.78\,\text{J} = -\frac{1}{2}(252\,\text{N/m})y^2 - (0.263\,\text{kg})(9.81\,\text{m/s}^2)y,$$

with solution $y = 0.225$ m.

P11-21 (a) We can solve this with a trick of integration.

$$
\begin{aligned}
W &= \int_0^x F\,dx, \\
&= \int_0^x ma_x\frac{dt}{dt}\,dx = ma_x\int_0^t \frac{dx}{dt}dt \\
&= ma_x\int_0^t v_x\,dt = ma_x\int_0^t at\,dt, \\
&= \frac{1}{2}ma_x^2 t^2.
\end{aligned}
$$

Basically, we changed the variable of integration from x to t, and then used the fact the the acceleration was constant so $v_x = v_{0x} + a_x t$. The object started at rest so $v_{0x} = 0$, and we are given in the problem that $v_\text{f} = at_\text{f}$. Combining,

$$W = \frac{1}{2}ma_x^2 t^2 = \frac{1}{2}m\left(\frac{v_\text{f}}{t_\text{f}}\right)^2 t^2.$$

(b) Instantaneous power will be the derivative of this, so

$$P = \frac{dW}{dt} = m\left(\frac{v_\text{f}}{t_\text{f}}\right)^2 t.$$

P11-22   (a) $\alpha = (-39.0 \text{ rev/s})(2\pi \text{ rad/rev})/(32.0 \text{ s}) = -7.66 \text{ rad/s}^2$.
   (b) The total rotational inertia of the system about the axis of rotation is

$$I = (6.40 \text{ kg})(1.20 \text{ m})^2/12 + 2(1.06 \text{ kg})(1.20 \text{ m}/2)^2 = 1.53 \text{ kg} \cdot \text{m}^2.$$

The torque is then $\tau = (1.53 \text{ kg} \cdot \text{m}^2)(7.66 \text{ rad/s}^2) = 11.7 \text{ N} \cdot \text{m}$.
   (c) $K = \frac{1}{2}(1.53 \text{ kg} \cdot \text{m}^2)(245 \text{ rad/s})^2 = 4.59 \times 10^4 \text{J}$.
   (d) $\theta = \omega_{\text{av}}t = (39.0 \text{ rev/s}/2)(32.0 \text{ s}) = 624$ rev.
   (e) Only the loss in kinetic energy is independent of the behavior of the frictional torque.

P11-23   The wheel turn with angular speed $\omega = v/r$, where r is the radius of the wheel and v the speed of the car. The total rotational kinetic energy in the four wheels is then

$$K\text{r} = 4\frac{1}{2}I\omega^2 = 2\left[\frac{1}{2}(11.3 \text{ kg})r^2\right]\left[\frac{v}{r}\right]^2 = (11.3 \text{ kg})v^2.$$

The translational kinetic energy is $K_{\text{t}} = \frac{1}{2}(1040 \text{ kg})v^2$, so the fraction of the total which is due to the rotation of the wheels is

$$\frac{11.3}{520 + 11.3} = 0.0213 \text{ or } 2.13\%.$$

P11-24   (a) Conservation of angular momentum: $\omega_{\text{f}} = (6.13 \text{ kg} \cdot \text{m}^2/1.97 \text{ kg} \cdot \text{m}^2)(1.22 \text{ rev/s}) = 3.80 \text{ rev/s}$.
   (b) $K_{\text{r}} \propto I\omega^2 \propto l^2/I$, so

$$K_{\text{f}}/K_{\text{i}} = I_{\text{i}}/I_{\text{f}} = (6.13 \text{ kg} \cdot \text{m}^2)/(1.97 \text{ kg} \cdot \text{m}^2) = 3.11.$$

| P11-25 | We did the first part of the solution in Ex. 10-21. The initial kinetic energy is (again, ignoring the shaft),

$$K_{\text{i}} = \frac{1}{2}I_1\vec{\omega}_{1,\text{i}}{}^2,$$

since the second wheel was originally at rest. The final kinetic energy is

$$K_{\text{f}} = \frac{1}{2}(I_1 + I_2)\vec{\omega}_{\text{f}}{}^2,$$

since the two wheels moved as one. Then

$$\begin{aligned}
\frac{K_{\text{i}} - K_{\text{f}}}{K_{\text{i}}} &= \frac{\frac{1}{2}I_1\vec{\omega}_{1,\text{i}}{}^2 - \frac{1}{2}(I_1 + I_2)\vec{\omega}_{\text{f}}{}^2}{\frac{1}{2}I_1\vec{\omega}_{1,\text{i}}{}^2}, \\
&= 1 - \frac{(I_1 + I_2)\vec{\omega}_{\text{f}}{}^2}{I_1\vec{\omega}_{1,\text{i}}{}^2}, \\
&= 1 - \frac{I_1}{I_1 + I_2},
\end{aligned}$$

where in the last line we substituted from the results of Ex. 10-21.
   Using the numbers from Ex. 10-21,

$$\frac{K_{\text{i}} - K_{\text{f}}}{K_{\text{i}}} = 1 - \frac{(1.27 \text{ kg} \cdot \text{m}^2)}{(1.27 \text{ kg} \cdot \text{m}^2) + (4.85 \text{ kg} \cdot \text{m}^2)} = 79.2\%.$$

P11-26 See the solution to P10-11.

$$K_i = \frac{I}{2}\omega^2 + \frac{m}{2}v^2$$

while

$$K_f = \frac{1}{2}(I + mR^2)\omega_f{}^2$$

according to P10-11,

$$\omega_f = \frac{I\omega - mvR}{I + mR^2}.$$

Then

$$K_f = \frac{1}{2}\frac{(I\omega - mvR)^2}{I + mR^2}.$$

Finally,

$$\begin{aligned} \Delta K &= \frac{1}{2}\frac{(I\omega - mvR)^2 - (I + mR^2)(I\omega^2 + mv^2)}{I + mR^2}, \\ &= -\frac{1}{2}\frac{ImR^2\omega^2 + 2mvRI\omega + Imv^2}{I + mR^2}, \\ &= -\frac{Im}{2}\frac{(R\omega + v)^2}{I + mR^2}. \end{aligned}$$

P11-27 See the solution to P10-12.

(a) $K_i = \frac{1}{2}I_i\omega_i{}^2$, so

$$K_i = \frac{1}{2}\left(2(51.2\,\text{kg})(1.46\,\text{m})^2\right)(0.945\ \text{rad/s})^2 = 97.5\,\text{J}.$$

(b) $K_f = \frac{1}{2}\left(2(51.2\,\text{kg})(0.470\,\text{m})^2\right)(9.12\ \text{rad/s})^2 = 941\,\text{J}$. The energy comes from the work they do while pulling themselves closer together.

P11-28 $K = \frac{1}{2}mv^2 = \frac{1}{2m}p^2 = \frac{1}{2m}\vec{\mathbf{p}}\cdot\vec{\mathbf{p}}$. Then

$$\begin{aligned} K_f &= \frac{1}{2m}(\vec{\mathbf{p}}_i + \Delta\vec{\mathbf{p}})\cdot(\vec{\mathbf{p}}_i + \Delta\vec{\mathbf{p}}), \\ &= \frac{1}{2m}\left(p_i{}^2 + 2\vec{\mathbf{p}}_i\cdot\Delta\vec{\mathbf{p}} + (\Delta p)^2\right), \\ \Delta K &= \frac{1}{2m}\left(2\vec{\mathbf{p}}_i\cdot\Delta\vec{\mathbf{p}} + (\Delta p)^2\right). \end{aligned}$$

In all three cases $\Delta p = (3000\,\text{N})(65.0\,\text{s}) = 1.95\times10^5\,\text{N}\cdot\text{s}$ and $p_i = (2500\,\text{kg})(300\,\text{m/s}) = 7.50\times10^5\,\text{kg}\cdot\text{m/s}$.

(a) If the thrust is backward (pushing rocket forward),

$$\Delta K = \frac{+2(7.50\times10^5\text{kg}\cdot\text{m/s})(1.95\times10^5\text{N}\cdot\text{s}) + (1.95\times10^5\text{N}\cdot\text{s})^2}{2(2500\,\text{kg})} = +6.61\times10^7\text{J}.$$

(b) If the thrust is forward,

$$\Delta K = \frac{-2(7.50\times10^5\text{kg}\cdot\text{m/s})(1.95\times10^5\text{N}\cdot\text{s}) + (1.95\times10^5\text{N}\cdot\text{s})^2}{2(2500\,\text{kg})} = -5.09\times10^7\text{J}.$$

(c) If the thrust is sideways the first term vanishes,

$$\Delta K = \frac{+(1.95\times10^5\text{N}\cdot\text{s})^2}{2(2500\,\text{kg})} = 7.61\times10^6\text{J}.$$

P11-29 There's nothing to integrate here! Start with the work-energy theorem

$$
\begin{aligned}
W &= K_\mathrm{f} - K_\mathrm{i} = \frac{1}{2}mv_\mathrm{f}{}^2 - \frac{1}{2}mv_\mathrm{i}{}^2, \\
&= \frac{1}{2}m\left(v_\mathrm{f}{}^2 - v_\mathrm{i}{}^2\right), \\
&= \frac{1}{2}m\left(v_\mathrm{f} - v_\mathrm{i}\right)\left(v_\mathrm{f} + v_\mathrm{i}\right),
\end{aligned}
$$

where in the last line we factored the difference of two squares. Continuing,

$$
\begin{aligned}
W &= \frac{1}{2}\left(mv_\mathrm{f} - mv_\mathrm{i}\right)\left(v_\mathrm{f} + v_\mathrm{i}\right), \\
&= \frac{1}{2}\left(\Delta p\right)\left(v_\mathrm{f} + v_\mathrm{i}\right),
\end{aligned}
$$

but $\Delta p = J$, the impulse. That finishes this problem.

P11-30 Let M be the mass of the helicopter. It will take a force Mg to keep the helicopter airborne. This force comes from pushing the air down at a rate $\Delta m/\Delta t$ with a speed of v; so $Mg = v\Delta m/\Delta t$. The blades sweep out a cylinder of cross sectional area A, so $\Delta m/\Delta t = \rho A v$. The force is then $Mg = \rho A v^2$; the speed that the air must be pushed down is $v = \sqrt{Mg/\rho A}$. The minimum power is then

$$
P = Fv = Mg\sqrt{\frac{Mg}{\rho A}} = \sqrt{\frac{(1820\,\mathrm{kg})^3(9.81\,\mathrm{m/s^2})^3}{(1.23\,\mathrm{kg/m^3})\pi(4.88\,\mathrm{m})^2}} = 2.49\times10^5\mathrm{W}.
$$

P11-31 (a) Inelastic collision, so $v_\mathrm{f} = mv_\mathrm{i}/(m+M)$.
(b) $K = \frac{1}{2}mv^2 = p^2/2m$, so

$$
\frac{\Delta K}{K_\mathrm{i}} = \frac{1/m - 1/(m+M)}{1/m} = \frac{M}{m+M}.
$$

P11-32 Inelastic collision, so

$$
v_\mathrm{f} = \frac{(1.88\,\mathrm{kg})(10.3\,\mathrm{m/s}) + (4.92\,\mathrm{kg})(3.27\,\mathrm{m/s})}{(1.88\,\mathrm{kg}) + (4.92\,\mathrm{kg})} = 5.21\,\mathrm{m/s}.
$$

The loss in kinetic energy is

$$
\Delta K = \frac{(1.88\,\mathrm{kg})(10.3\,\mathrm{m/s})^2}{2} + \frac{(4.92\,\mathrm{kg})(3.27\,\mathrm{m/s})^2}{2} - \frac{(1.88\,\mathrm{kg} + 4.92\,\mathrm{kg})(5.21\,\mathrm{m/s})^2}{2} = 33.7\,\mathrm{J}.
$$

This change is because of work done on the spring, so

$$
x = \sqrt{2(33.7\,\mathrm{J})/(1120\,\mathrm{N/m})} = 0.245\,\mathrm{m}
$$

P11-33 $\vec{\mathbf{p}}_{\mathrm{f},B} = \vec{\mathbf{p}}_{\mathrm{i},A} + \vec{\mathbf{p}}_{\mathrm{i},B} - \vec{\mathbf{p}}_{\mathrm{f},A}$, so

$$
\begin{aligned}
\vec{\mathbf{p}}_{\mathrm{f},B} &= [(2.0\,\mathrm{kg})(15\,\mathrm{m/s}) + (3.0\,\mathrm{kg})(-10\,\mathrm{m/s}) - (2.0\,\mathrm{kg})(-6.0\,\mathrm{m/s})]\hat{\mathbf{i}} \\
&\quad + [(2.0\,\mathrm{kg})(30\,\mathrm{m/s}) + (3.0\,\mathrm{kg})(5.0\,\mathrm{m/s}) - (2.0\,\mathrm{kg})(30\,\mathrm{m/s})]\hat{\mathbf{j}}, \\
&= (12\,\mathrm{kg}\cdot\mathrm{m/s})\hat{\mathbf{i}} + (15\,\mathrm{kg}\cdot\mathrm{m/s})\hat{\mathbf{j}}.
\end{aligned}
$$

Then $\vec{v}_{f,B} = (4.0\,\text{m/s})\hat{\mathbf{i}} + (5.0\,\text{m/s})\hat{\mathbf{j}}$. Since $K = \frac{m}{2}(v_x^2 + v_y^2)$, the change in kinetic energy is

$$\begin{aligned}\Delta K &= \frac{(2.0\,\text{kg})[(-6.0\,\text{m/s})^2 + (30\,\text{m/s})^2 - (15\,\text{m/s})^2 - (30\,\text{m/s})^2]}{2} \\ &\quad + \frac{(3.0\,\text{kg})[(4.0\,\text{m/s})^2 + (5\,\text{m/s})^2 - (-10\,\text{m/s})^2 - (5.0\,\text{m/s})^2]}{2} \\ &= -315\,\text{J}.\end{aligned}$$

P11-34 For the observer on the train the acceleration of the particle is a, the distance traveled is $x_\text{t} = \frac{1}{2}at^2$, so the work done as measured by the train is $W_\text{t} = max_\text{t} = \frac{1}{2}a^2t^2$. The final speed of the particle as measured by the train is $v_\text{t} = at$, so the kinetic energy as measured by the train is $K = \frac{1}{2}mv^2 = \frac{1}{2}m(at)^2$. The particle started from rest, so $\Delta K_\text{t} = W_\text{t}$.

For the observer on the ground the acceleration of the particle is a, the distance traveled is $x_\text{g} = \frac{1}{2}at^2 + ut$, so the work done as measured by the ground is $W_\text{g} = max_\text{g} = \frac{1}{2}a^2t^2 + maut$. The final speed of the particle as measured by the ground is $v_\text{g} = at + u$, so the kinetic energy as measured by the ground is

$$K_\text{g} = \frac{1}{2}mv^2 = \frac{1}{2}m(at + u)^2 = \frac{1}{2}a^2t^2 + maut + \frac{1}{2}mu^2.$$

But the initial kinetic energy as measured by the ground is $\frac{1}{2}mu^2$, so $W_\text{g} = \Delta K_\text{g}$.

P11-35 (a) $K_\text{i} = \frac{1}{2}m_1v_{1,\text{i}}{}^2$.

(b) After collision $v_\text{f} = m_1v_{1,\text{i}}/(m_1 + m_2)$, so

$$K_\text{f} = \frac{1}{2}(m_1 + m_2)\left(\frac{m_1v_{1,\text{i}}}{m_1 + m_2}\right)^2 = \frac{1}{2}m_1v_{1,\text{i}}{}^2\left(\frac{m_1}{m_1 + m_2}\right).$$

(c) The fraction lost was

$$1 - \frac{m_1}{m_1 + m_2} = \frac{m_2}{m_1 + m_2}.$$

(d) Note that $v_\text{cm} = v_\text{f}$. The initial kinetic energy of the system is

$$K_\text{i} = \frac{1}{2}m_1v'_{1,\text{i}}{}^2 + \frac{1}{2}m_2v'_{2,\text{i}}{}^2.$$

The final kinetic energy is zero (they stick together!), so the fraction lost is 1. The *amount* lost, however, is the same.

P11-36 Only consider the first two collisions, the one between m and m', and then the one between m' and M.

Momentum conservation applied to the first collision means the speed of m' will be between $v' = mv_0/(m + m')$ (completely inelastic) and $v' = 2mv_0/(m + m')$ (completely elastic). Momentum conservation applied to the second collision means the speed of M will be between $V = m'v'/(m' + M)$ and $V = 2m'v'/(m' + M)$. The largest kinetic energy for M will occur when it is moving the fastest, so

$$v' = \frac{2mv_0}{m + m'} \text{ and } V = \frac{2m'v'}{m' + M} = \frac{4m'mv_0}{(m + m')(m' + M)}.$$

We want to maximize V as a function of m', so take the derivative:

$$\frac{dV}{dm'} = \frac{4mv_0(mM - m'^2)}{(m' + M)^2(m + m')^2}.$$

This vanishes when $m' = \sqrt{mM}$.